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ℓ-Interval Parking Functions

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Abstract of Report Talk: Parking functions of length n, vectors representing the number of ways n cars can park on a one way street, are a well-studied combinatorial object. One way to generalise them is to consider each car as parking, at most, a fixed interval ℓ away from their preference. We call these ℓ-interval parking functions. Scholar Kimberly P. Hadaway has shown that, when $\ell = 1$, these functions are in bijection with the Fubini rankings of the same length. In this talk, we expand upon her work to present a generalised recursive formula for when ℓ is any natural number. Additionally, we present formulae for the number of nondecreasing ℓ-interval parking functions, while also finding new and interesting connections to objects such as the Fubini rankings, Dyck paths, and the Fibonacci numbers. **[AT05175949]**

[Joint with Dr. Rebecca Garcia, Dr. Pamela E. Harris, and Kobe Lawson-Chavanu] Received: August 17, 2021

A Bijection between Parking Functions and The Tower of Hanoi

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Abstract of Report Talk: Parking functions with "bumps" have a total of d spaces between the desired parking spots of n cars and the spots in which they actually park. In a previous study by Harris, Kotapati, and Saylor, the set of parking functions of length n with exactly one bump was found to be equinumerous to the set of distinct states in the famous combinatorial puzzle, the Tower of Hanoi, with $n + 1$ disks and $n + 1$ pegs. In our research, we created two functions that illustrate an explicit bijection between the two sets, proving an open problem stated in their work. **[AY05211001]**

[Joint with Casandra Monroe, Dr. Pamela E. Harris, Camelle Tieu, Surya Kotapati, and Zia Saylor] Received: August 18, 2021

A Dense Wild Two Point Algebra

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Abstract of Report Talk: Arbitrary finite dimensional K-algebras have been classified into three types by the famous theorem of Drozd by their representation type. These categories are Finite, Tame, and Wild. Our focus is on Algebras of Wild Representation type. As the name suggest, the indecomposable representations are not feasible to classify. So, they are indeed wild. However, Dense Orbit Algebras that are of wild representation type are conjectured to have finitely many representations, up to isomorphism classes, that are dense in their orbits. Hence the classification of these representations becomes feasible. There are very few examples of such algebras in the current literature. We explore a particular two algebra of wild representation type and show, with the help of a computer program created specifically for this mathematical problem which simplifies a given variable matrix, that the algebra is Dense Orbit Algebra. **[AA04125807]**

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Computations of the involutive concordance invariants of (1,1)-knots

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Abstract of Report Talk: A knot $K \subseteq S^3$ is a smooth embedding $S^1 \hookrightarrow S^3$; two knots $K_1, K_2 \subseteq S^3$ are said to be concordant if they form the boundary of an annulus properly embedded into $S^3 \times [0,1]$. Heegaard Floer knot homology is an invariant of knots introduced by P. Ozsváth and Z. Szabó in the early 2000's which associates to a knot K a $(\mathbb{Z} \oplus \mathbb{Z})$-filtered, $\mathbb{Z}$-graded chain complex over $\mathbb{F}_2[U, U^{-1}]$ called $CFK^\infty(K)$, and improves on classical invariants of the knot. Involutive Heegaard Floer homology is a variant theory introduced in 2015 by K. Hendricks and C. Manolescu which additionally considers a chain map on $CFK^\infty(K)$ called ι_K, and extracts from this data two new numerical invariants of knot concordance, $\underline{V}_0(K)$ and $\overline{V}_0(K)$. These new invariants are interesting, because, unlike other concordance invariants from Heegaard Floer homology, they do not necessarily vanish on knots of finite order in the group of concordance classes of knots. The map ι_K is in general difficult to compute, and computations have been carried out for relatively few knots. We give a complete computation of ι_K for 10 and 11-crossing knots satisfying a certain simplicity condition, called the $(1,1)$-knots. Our methods are principally focused on homological algebra. **[AA05181958]**

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One-Gap Solution to the Kaup-Broer System

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Abstract of Report Talk: We use elliptic function theory to derive one-gap solutions to the Kaup-Broer (KB) system, a coupled system of nonlinear partial differential equations. Similar to the Korteweg de Vries (KdV) equation, this system describes shallow water waves with weakly nonlinear restoring forces. In our paper, we compare the utility and methods for finding solutions to each of these models as well as provide a general overview of nonlinear wave theory, which is far more accurate at predicting tsunami and rogue wave amplitude than linear wave theory. In both the KB and KdV systems, we generate solutions using the Weierstrass $\wp$-function demonstrating the underlying algebraic geometry. Particularly, the construction of a birational transformation between elliptic curves is necessary for finding such solutions to the KB system. In future work, we plan to extend our results using hyper-elliptic curves of genus g, where we intend to investigate what happens as $g \to \infty$. **[AA06153258]**

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The Fairy Bread Sandwich Theorem

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Abstract of Report Talk: The Ham Sandwich theorem is a classic result about partitions of measures in Euclidean space and one of the first applications of topology to combinatorial geometry. We use recent extensions of the Borsuk-Ulam theorem to generalize the Ham Sandwich theorem. A k-dimensional mass assignment continuously imposes a measure on each k-dimensional affine subspace of $\mathbb{R}^d$. Given a finite collection of mass assignments in each dimension, one may ask if there is some sequence of affine subspaces $S_{k-1} \subset ... \subset S_{d-1} \subset \mathbb{R}^d$ such that S_i bisects all the mass assignments on S_{i+1} for every i. We show it is possible to do so whenever naive dimension counting allows. This improves the numbers given by a simple application of the Ham Sandwich theorem. We use this result to prove several conjectures by Schnider regarding mass assignments. **[AI02224151]**

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On the Solvability of a Bipartite Graph Reduction Game

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Moravian University [Mentor:Kathleen Ryan]

Abstract of Report Talk: A *game-labeling* of a bipartite graph is one in which the vertices have non-negative labels and the label sums of the partite sets are equal. A *reduction across an edge* is one in which the endpoints are both reduced by the same integral amount. We introduce a single player game on a game-labeled bipartite graph G where each move is a reduction across an edge and no move produces a negative label. The goal of the game is to reduce all labels in G to 0, and if the player succeeds in doing so, the player wins the game. In this presentation, we present a necessary and sufficient sum condition for detecting the solvability of the game. We also demonstrate the connection between this game and Double Choco Puzzles. **[BK29112702]**

[Joint with Garrison Koch, Kyla Shappell]
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Betti tables of quadratic ideals in characteristic two

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Abstract of Report Talk: In commutative ring theory, the Betti table of an ideal of a graded ring consists of numerical data called graded Betti numbers arranged in tabular form. Betti numbers quantify the minimal number of generators required to generate the ideal, the minimal number of generators for the set of relations on the generators of the ideal, and so on. Quadratic ideals have especially intriguing Betti tables, which have been determined in some cases under the assumption that the characteristic is different from two.

We examine the Betti tables of quadratic ideals with three generators in the polynomial ring $\mathbb{F}_2[x, y, z]$ of characteristic two. The set of all such ideals can be divided into four categories based on certain subsets of the generating set forming regular sequences. With the help of computations (using the computer algebra system Macaulay2), and applying certain results on Koszul complexes and mapping cones, we manage to resolve almost all such ideals. Several of our results hold in general polynomial rings. We also provide efficient methods to compute all possible Betti tables of quadratic ideals in $\mathbb{F}_2[x, y, z]$, the number of quadratic ideals corresponding to each possible Betti table and the Haase diagrams of Betti sequences corresponding to ideals of codimension 1, 2, and 3.

This work was completed in the 2021 Polymath Jr. program. **[BM05160504]**

[Joint with Arav Agarwal, William DeGroot, Youngmin Ko, Zachary Linn, Huiwen Lu, Xinyi Lu, Atharva Pradhan, Samuel Rath]
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The Numerical Range and Spectrum of a Composition Operator induced by an Anti-Diagonal Matrix

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Abstract of Report Talk: We consider the numerical ranges and spectrums of composition operators C_A induced by anti-diagonal matrices A on the Hardy space $H^2(\mathbb{B}_n)$ on the open unit ball $\mathbb{B}_n$ in $\mathbb{C}^n$. In the case when $\|A\| < 1$, we show that the spectrum of C_A is the set of all possible products of the eigenvalues of A and the point $\{0\}$. The key to this calculation is showing the subspace of homogeneous polynomials of degree k, H_k, is a reducing subspace and then showing the composition operator restricted to H_k is unitarily equivalent to an orthogonal direct sum of 2 by 2 matrices.

We characterize the numerical range of C_A as the convex hull of an infinite union of elliptical disks whose foci are points in the spectrum of C_A. We determine sufficient conditions for the numerical range of C_A to be equal to the finite union of the elliptical disks. We prove, when A is an n by n anti-diagonal matrix which is a self-map of $\mathbb{B}_n$, that the boundary of the numerical range of C_A intersects the unit circle if and only if C_A has a reducing subspace on which C_A is unitary. **[BT05175457]**

[Joint with Lauren Forbes]

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Counting Unit Simplices in $\mathbb{R}^d$

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Abstract of Report Talk: A *unit k-simplex* is a set of k points in $\mathbb{R}^d$ with pairwise unit distances between them. An interesting problem in Discrete Geometry asks for the maximum possible number of unit k-simplices that can be determined by a set of n points in $\mathbb{R}^d$. For $d \geq 4$ and $1 \leq k \leq d/2$, it is not very hard to show that the answer to this question is of the order n^k. However, for $d/2 < k < d+1$ the problem is wide open for most cases. Erdős and Purdy conjectured that for that setting the answer is of the order $n^{d/2}$. The conjecture is motivated by constructions that match this bound. We make progress towards this conjecture by confirming it for the case of $k = d/2 + c$ for any constant c and sufficiently large $d = d(c)$. Our proof builds on the work of Agarwal and Sharir; using classical cutting techiques, one can reduce the problem to solving a large family of geometrically defined linear optimisation problems. By a careful analysis we solve the corresponding linear optimisations to achieve our result. **[BE05215745]**

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Invariant Geometric Structures on Complex Almost Abelian Groups

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Abstract of Report Talk: An almost Abelian group is a non-Abelian Lie group with a codimension 1 Abelian subgroup. This project investigates invariant Hermitian and Kähler structures on complex almost Abelian groups. In doing so, we find explicit formulas for the left and right Haar measures, the modular function, and left and right generator vector fields on simply connected complex almost Abelian groups. From the generator fields, we obtain invariant vector and tensor field frames, allowing us to find an explicit form for all invariant tensor fields. Namely, all such invariant tensor fields have constant coefficients in the invariant frame. From this, we classify all invariant Hermitian metrics on complex simply connected almost Abelian groups, and we prove the nonexistence of invariant Kähler metrics on all such groups. Via constructions involving the pullback of the quotient map, we extend the explicit description of invariant Hermitian metrics and the nonexistence of Kähler metrics to quotients of complex simply connected almost Abelian groups. **[BO05163305]**

[Joint with Jimmy Morentin and Tianyi Wang]

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Unknotting Number of Cabled Twist Knots

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Abstract of Report Talk: A knot is an embedding of S^1 into $\mathbb{R}^3$, a smooth closed loop in 3-space that does not intersect itself. The unknotting number of a knot is the minimum number of crossing changes needed to undo a knot into a circle. The unknotting number is a basic invariant used to measure the complexity of a knot, but in general it is hard to compute. However, there are lower bounds for the unknotting number from modern homological knot invariants, such as Floer homology. The satellite operation of a knot P onto K given by $P(K)$ is done by placing P into a torus and transforming the torus into the knot K giving the knot $P(K)$. We study the unknotting number for families of satellite knots. We find upper bounds for their unknotting numbers by introducing an unknotting process for them. A family of knots we study are the cables of twist knots. Twist knots are obtained by linking the ends of a twisted loop. Cables are formed by the satellite operation of a torus knot onto a knot K. One of the unknotting number lower bounds from knot Floer homology, defined by Ozsváth and Szabó and independently Rasmussen, is $|\tau|$. We use Hom's formula to compute it for cables of twist knots. For an infinite subfamily of our knots we reach an exact value for the unknotting number. In this talk after reviewing the basic definitions, we will present unknotting procedures for various satellite knots then focus primarily on cables of twist knots. We describe the lower and upper bounds alongside their differences to give criteria for when the bounds match and when they fall within a certain range of each other. **[BD06132646]**

[Joint with Melissa Zhang]

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Recurrences of Small Divisors

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Abstract of Report Talk: Given a positive integer n, the small divisors of n are defined as the positive divisors of n that do not exceed $\sqrt{n}$. Ianucci previously classified all n for which the small divisors of n form an arithmetic progression. We classify all n for which the small divisors of n form a linear recurrence of order at most two. **[CA06163558]**

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Enumerating Polyominoes on the Torus and Other Finite Surfaces

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Abstract of Report Talk: Since their popularization in the mid-20th century, significant attention has been directed to the counting of polyominoes on an infinite plane. A polyomino is a shape consisting of unit squares that are connected by their edges. In our research, we pursue the less examined problem of counting polyominoes on finite surfaces with different dimensions, such as the torus and Möbius strip. Specifically, we examine the fixed polyominoes, where different orientations of a polyomino are considered distinct. We first perform by-hand enumeration for surfaces of smaller dimensions to learn more about the properties of polyominoes on each surface. Utilizing existing algorithms as a foundation, we then employ computational methods to enumerate and count polyominoes for larger cases. Currently, we have counted up to the 19-ominoes on the 6×6 torus, cylinder, and finite grid. We have also found examples of polyominoes that exist on the torus but not on the finite grid and analyzed the conditions for this to occur. This presentation will summarize the results of the CC-REU NSF summer REU experience (DMS-2050692) where these questions were explored. **[CW05203618]**

[Joint with Naftoli Kolodny, Jonny Quezada] Received: August 18, 2021

Subsums of Random Numbers

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Abstract of Report Talk: If you pick n random numbers in $[0,1]$, what is the probability that their sum also falls into the interval $[0,1]$? The answer turns out to be $1/n!$, which can be seen using symmetry arguments or multidimensional integrals. In our research, we consider more generally the probability, $p(n,k)$, that, given n random numbers in $[0,1]$, there are k of these numbers whose sum falls into the interval $[0,1]$.

We prove the explicit formula

$$p(n,k) = 1 + \frac{1}{(k-1)!}\sum_{i=1}^{k-1}(-1)^{k-i}\cdot\binom{k-1}{i}\cdot i^n\cdot(i+1)^{-n+k-1}.$$

We also show that $p(n,k)$ satisfies the recurrence

$$p(n,k) = \frac{1}{k}\cdot p(n-1,k-1) + \frac{k-1}{k}\cdot p(n-1,k)$$

and has generating function

$$\sum_{n=k}^{\infty} p(n,k)x^k = \frac{x^k}{k!\cdot(1-x)(1-\frac{1}{2}x)\cdots(1-\frac{k-1}{k}x)}.$$

These formulas are analogous to formulas for Stirling numbers of the second kind, $S(n,k)$, which count the number of ways to partition an n-element set into k nonempty subsets.
[CH04235953]

[Joint with Wei Wang]

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The Krein–von Neumann Extension of a Regular Even Order Quasi-Differential Operator

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Abstract of Report Talk: We characterize by boundary conditions the Krein–von Neumann extension of a strictly positive minimal operator corresponding to a regular even order quasi-differential expression of Shin–Zettl type. The characterization is stated in terms of a specially chosen basis for the kernel of the maximal operator and employs a characterization of the Friedrichs extension due to Möller and Zettl. Using this characterization, we recover results on the Krein–von Neumann extension of the Sturm-Liouville operator by Clark et al. and Eckhardt et al. as well as of the pure differential operator by Granovskyi and Oridoroga. **[CM02155432]**

[Joint with Brian Udall] Received: August 17, 2021

The vulnerability of the Gale and Shapley's algorithm with required truncation

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Abstract of Report Talk: Gale and Shapley's deferred acceptance algorithm is widely applied in real-life such as in medical residency matching, school choice, and the assignment of cadets to military branches. In theory, Gale and Shapley's algorithm is strategy-proof—no agent can distort the match outcome to their advantage by reporting false preferences. However, in practice, Gale and Shapley's algorithm may not be strategy-proof because of the required truncation of the preference lists. We compare the manipuability of Gale and Shapley's algorithm with fixed truncation over variations in the sets of students, schools, preferences, or quotas. **[CY06125907]**

[Joint with Shira Li] Received: August 17, 2021

Symmetries of linear systems on graphs

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Abstract of Report Talk: In the divisor theory of graphs, a finite, connected graph is viewed as a discrete analog of a Riemann surface. A divisor D on a graph is an assignment of integers to each vertex of the graph. An important statistic in this setting is the complete linear system of D, which is the collection of effective (non-negative) divisors linearly equivalent to D via the discrete Laplacian operator. Recently, S. Brauner, F. Glebe, and D. Perkinson characterized all complete linear systems on a finite graph G using a system of generating sets, each of which correspond to a cone. We extend their results by using generating sets to compute the subset of effective divisors fixed by a symmetry γ of G, as well as the subset of effective divisors that are stable under action of γ. In certain cases, we are able to extend the action of γ to an action on the cones arising from these generating sets. **[CA05141419]**

[Joint with Nhung Pham]
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Counting Acyclic Orientations of Signed Graphs

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Abstract of Report Talk: In 1995, Richard Stanley introduced the symmetric chromatic function of a graph, which is a power series in the variables $x_1, x_2, \ldots$ He then proved that when this function is written in a certain basis, the coefficients reveal information about the acyclic orientations of the graph.

The goal of our research is to generalize Stanley's result to signed graphs. A signed graph is a graph whose edges are labeled with either a plus sign or a minus sign. The definitions of graph colorings and graph orientations can be naturally modified to interact with signed edges.

Groups from previous years of the same research program successfully developed analogous symmetric chromatic functions in the variables $\ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots$ for signed graphs, and we have fully generalized Stanley's theorem to signed graphs by finding an appropriate basis set for these new functions. This result is comparable in simplicity to the 1995 theorem. **[CO06154626]**

[Joint with Jake Huryn]
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Deep Learning Techniques for Solving Semilinear Parabolic Partial Differential Equations

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Abstract of Report Talk: Problems in physics, engineering, finance, and many other fields require solving partial differential equations (PDE). It is preferable to get a precise analytical solution to PDEs; however, many cannot be solved exactly using analytical methods. This has led to the creation of many numerical algorithms to solve the problems for which analytical solutions cannot be derived. Deep learning techniques can be useful for high dimensional PDEs where traditional numerical methods such as finite difference or finite element methods fail due to the curse of dimensionality. A backwards stochastic differential equation (BSDE) solver can be used to solve parabolic PDEs in both low and high dimensions. Euler's method is used to discretize the time space, and deep learning techniques are used to approximate changes in spatial variables. For semilinear parabolic PDEs, this approach enables the PDE to be solved numerically. Such an approach was used to investigate examples including the heat equation, a simple reaction-diffusion equation, and the Black-Scholes equation. The results were compared to analytical solutions and other numerical methods. The numerical results show that the BSDE solver is capable of numerically solving PDEs in high dimensions, and also comparable to traditional numerical techniques in low dimensions in terms of accuracy.
[DE06163812]

[Joint with Kalani Rubasinghe, Lizzy Javor, Luis Topete] Received: August 18, 2021

Reducibility of Sets in Generalized Settings

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Abstract of Report Talk: Sumsets have applications across fields: they are a powerful combinatorial tool in studying Erdos distance problems in discrete geometry; they form the core of the arithmetic Kakeya conjecture, which would imply a form of the Kakeya conjecture in harmonic analysis; and they were central to the proof of Szemeredi's Theorem, a core result in arithmetic combinatorics. Given a set S, we are interested in the question of whether or not S can be written non-trivially as the sum of two sets. If there are sets A, B with $|A|, |B| \geq 2$ such that $S = A+B$, we call S *reducible*. If S is not reducible, we call S *irreducible*. We prove results about the reducibility of subsets of $[n]^d$ for $d \geq 2$ and the reducibility of the so-called lunar numbers, a generalization of the subsets of $[n]$.

In 2011, Applegate, Le Brun, and Sloane first described *lunar arithmetic*, a system for addition and multiplication without carries. We can draw a correspondence between base 2 lunar numbers and subsets of the natural numbers. Under this correspondence, addition of sets is equivalent to multiplication of lunar numbers, allowing us prove results about the reducibility of sets through investigation of lunar operations. This correspondence between base 2 lunar multiplication and set addition also allows us to understand base b lunar multiplication as a generalization of set addition.

Applegate, Le Brun, and Sloane conjectured that the number of base b lunar primes with k digits tends to $(b-1)^2 b^{k-2}$ as $k \to \infty$. A necessary and sufficient condition for a base-b lunar number to be prime is that it is *irreducible*, has a nonzero constant term, and contains the digit $b-1$. Their conjecture equivalently states that almost all lunar numbers satisfying the latter two conditions are irreducible. In 2014, Shitov showed that almost all boolean polynomials with non-zero constant term are irreducible, which implies that almost all base-2 lunar numbers with non-zero final digit are irreducible, confirming Applegate et. al.'s conjecture in the case $b = 2$. Generalizing Shitov's argument, we show that almost all lunar numbers are irreducible regardless of base. The conjecture of Applegate et al. then follows from this result.

We say that S is *asymptotically irreducible* if S is irreducible and making any finite number of changes to S results in an irreducible set. In 1953, Wirsing proved that almost all subsets of $\mathbb{N}$ are asymptotically irreducible. We generalize Wirsing's result by extending our notion of lunar numbers to include possibly infinitely long digit strings equipped with lunar operations. We define a notion of asymptotic irreducibility for such digit strings which in the $b = 2$ case agrees with the definition of asymptotic irreducibility for sets under the correspondence between base 2 lunar numbers and sets. Thus, Wiring's result implies that almost all digit strings composed of the digits 0 and 1 are asymptotically irreducible. We prove that almost all digit strings composed of digits in $\{0, 1, \ldots, b-1\}$ are asymptotically irreducible regardless of the choice of b. This result is thus a sort of infinite analog of our previous result, which stated that almost all lunar numbers of finite length are irreducible.

We have also obtained results pertaining to sets in higher dimensions. In particular, we define $[n] := \{0, 1, \ldots n\}$ and investigate subsets of $[n]^d$ for $d \geq 2$. We show that for a set $S \subseteq [n]^d$ with $k = \left|[n]^d \setminus S\right|$, S is reducible if

$$\frac{k}{d}\ln 2 + H_{\lceil \frac{k}{d} \rceil} + H_{\lceil \binom{k}{2}/d \rceil} < H_{n-1},$$

where H_ℓ is the ℓ-th Harmonic number. We also define a condition of *p-local irreducibility* and

Benfordness of the Riemann Mapping Function for the Reciprocal of the Mandelbrot Set

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Abstract of Report Talk: While one might initially conjecture that the distribution of nonzero leading digits is uniformly distributed, i.e., each digit appears $\approx 11\%$ of the time, many data sets tend to follow the logarithmic relation, with 1 as the leading digit $\approx 30\%$ of the time, 2 appearing $\approx 18\%$ of the time, and higher digits being successively less likely. Specifically the digits in base b are distributed according to the probability measure $\log_b\left(\frac{d+1}{d}\right)$ with $b \geq 2, 0 < d < b$. This distribution is known as Benford's Law, and it is ubiquitous, spanning across multiple orders of magnitude and fields. Interestingly, one area that has not been extensively studied in relation to Benford's Law is Fractal Geometry. We focused on Riemann Mapping of the exterior of Mandelbrot Set by looking at the coefficients of this Laurent Series. The asymptotic convergence of the series is intimately related to the conjectured local connectivity of the Mandelbrot Set. However, research on the conjecture has been slowed due to the sparse information concerning the coefficients, and no comprehensive lists exist for reference either.

Specifically we aimed to study the Benfordness of the Laurent Coefficients $a_{2,m}$ of the normalized Riemann mapping function: $f_2(z) = 1/\Psi_2(1/z) = z + \sum_{m=2}^{\infty} a_{2,m} z^m$ for the exterior of the reciprocal of the Mandelbrot set $\mathcal{R}_2 = \{1/z : z \in \mathcal{M}_2\}$. The coefficients are strictly binary rational numbers so we could study the distribution of the numerators, denominators, and decimal expansions independently. Computing the coefficients requires evaluating a finite series of Generalized Binomial Coefficients iterated over the solutions to a Linear Diophantine Equation. The time complexity grows exponentially as a result, but we were able to bypass some of the computation by looking at patterns in the solutions to the Diophantine Equations. The higher order solutions can be modified to obtain the solutions to all lower order terms, so it is possible to acquire solutions inductively. We used this top-down method to generate our coefficients, and then we performed statistical analysis to determine their distribution along with an analysis of their formulation to get a better understanding of their properties. There are very simple results from fractal sets such as the Cantor Set and the Sierpinski Triangle that show the unique lengths are Benford in most bases, we believe similar results hold for problems related to the Mandelbrot Set. **[DJ06163423]**

[Joint with Weike Fang]

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On the Minimal Generating Sets of Invariants for Finite Abelian Groups

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Abstract of Report Talk: Given some arbitrary polynomial ring, an invariant polynomial is a polynomial that is unchanged by the action of a group G. We investigate the ring of invariant polynomials under the action of some abelian groups with the goal of finding generators for this ring. When considering an abelian group, we can always find a basis such that the action is diagonal, so there exists monomial generators m_i for the invariant ring. By Noether's degree bound, the minimal set of generating monomials $\langle m_1, \ldots, m_k \rangle$ is finite and the degree of each generating monomial m_i is less than $|G|$. Motivated by the previous work of Gandini and Derksen, we present a new approach to find invariants for $\mathbb{Z}_p \oplus \mathbb{Z}_p$ and show that this approach can fail for $\mathbb{Z}_n \oplus \mathbb{Z}_n$ when n is not a prime. **[DM05180730]**

[Joint with Sam Ratliff]

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Colored Lattice Models and Colored Gelfand-Tsetlin Patterns

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Abstract of Report Talk: In searching for a bijective version of the proof presented in Weyl Group Multiple Dirichlet Series by Brubaker, Bump, and Friedberg, which states that two definitions of the p-parts of a multiple Dirichlet series given a Type A root system are equal, our group, mentored by Ben Brubaker himself, tackled defining new combinatorial objects: colored lattice models and their corresponding colored Gelfand-Tsetlin patterns. In this talk, we will discuss the problem, which is motivated by representation theory and number theory, how our definitions tackle it, and the progress and pitfalls we've encountered so far. **[DW06234018]**

[Joint with Andy Hardt, Alette Wells, Berwyn Prichard-Jones, Bryan Boehnke, Bowen Li, Cedric Gerdes, and others...]

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Numerical Simulations in Optimal Transport

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Abstract of Report Talk: In this project, we propose a numerical method to solve the classic L2-optimal transport problem. Optimal transport has a wide range of applications, including economics, machine learning, image processing, and more. Following the fluid dynamics reformulation of this problem in Benamou and Brenier (2000), we apply spatial discretization to solve the underlying partial differential equation. Our algorithm is based on the use of multiple shooting, in combination with a continuation procedure, to solve the boundary value problem derived from spatially discretizing this partial differential equation. We present several numerical examples to illustrate the effectiveness of the method. **[DS06140337]**

[Joint with Haomin Zhou, Jianbo Cui] Received: August 18, 2021

Dynamics of Vibro-Impact Energy Harvesters

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Georgia Tech [Mentor:Rachel Kuske]

Abstract of Report Talk: Energy Harvesting is a process of generating electrical energy from renewable sources. In particular, the subject of the research was a vibro-impact energy harvester (VI-EH), which is a capsule with a ball that harvests energy from vibrations. The case of VI-EH with symmetric restitution coefficients was considered by Kuske, Serdukova, and Yurchenko (2019). We built on this work by considering asymmetric coefficients for the harvester, development of a more realistic model, and examination of the dynamical behavior and the implications for energy output. We derived analytical results for the case with asymmetric restitution coefficients, and compared these with the numerical results obtained from varying the parameter of restitution coefficients on each end of the capsule. The results obtained were then used to provide suggestions for the best asymmetric restitution values for energy output. **[DS06145727]**

[Joint with Larissa Serdukova, Daniil Yurchenko] Received: August 18, 2021

Closed form densities for the limiting spectral measure of random circulant Hankel matrices.

Teresa M Dunn (mtdunn@ucdavis.edu)
()
Williams College [Mentor:Steven Miller]

Abstract of Report Talk: Random matrix theory was created to study the behavior of particles in nuclei as they follow Schrodinger's equation, $\hat{\mathrm{H}}\psi_n = E_n\psi_n$, where $\hat{\mathrm{H}}$ is an infinite dimensional matrix of unknown entries whose structure depends on the state of the physical system. To understand $\hat{\mathrm{H}}$, we study the eigenvalue distribution of families of random matrices. To compensate for the matrices being of finite dimension N, we study the limit as $N \to \infty$.

To formalize studying the distribution of the eigenvalues, we define the spectral measure for a given $N \times N$ matrix A as

$$\mu_{A,N}(x)\mathrm{d}x := \frac{1}{N}\sum_{i=1}^{N}\delta\left(x - \frac{\lambda_i(A)}{2\sqrt{N}}\right)\mathrm{d}x.$$

Integrating against this measure counts the number of normalized eigenvalues within any interval. By the Eigenvalue Trace Lemma, the kth moment of $\mu_{A,N}(x)$ corresponds to $\mathrm{tr}(A^k)$. The entries of the random matrix are i.i.d. random variables with mean 0, variance 1, and finite higher moments, facilitating study of the expectation of $\mathrm{tr}(A^k)$. The argument reduces to characterizing the terms in the trace formula that contribute in the limit, leading to combinatorial problems specific to each ensemble. Classical arguments prove that numerous nice families of matrices have a limiting spectral measure, but only rarely can it be written down in closed form.

In particular, we analyze the ensemble of circulant Hankel matrices. These entries satisfy $a_{ij} = a_{kl}$ if and only if $i + j \equiv_N k + l$. Within this ensemble, summands of $\mathrm{tr}(A^k)$ may be characterized via vectors of indices satisfying systems of equations of the form $i + j \equiv_N k + \ell$. The nullity of such index matrices then provides a bound on the degree of freedom of corresponding summands in the trace formula; vectors in the null space of the index matrices describe which indices correspond to equal entries of the circulant Hankel matrix. Using such combinatorial arguments, we completely characterize the summands of $\mathrm{tr}(A^k)$ and derive the moments of $\mu_{A,N}(x)$ as $N \to \infty$. We provide a new proof, using the method of moments, that the limiting spectral measure distribution converges almost surely to the Rayleigh distribution. We also introduce a new ensemble, concentric even matrices, generalize their properties to introduce a new matrix operation, and show that their limiting spectral distribution also converges almost surely to the Rayleigh distribution. **[DT03115401]**

[Joint with Henry Fleischmann, Faye Jackson, Simran Khunger, Luke Reifenberg, Stephen Willis] Received: August 17, 2021

Problems in Knot Theory and 4D Topology

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Abstract of Report Talk: *Knots* are smooth closed loops in 3-space that do not intersect themselves. They are considered *equivalent* when one can be smoothly deformed into another without passing through itself. The theory of knots has important applications in 3- and 4-dimensional topology. A knot is called *amphichiral* if it is equivalent to its mirror, and it is called *slice* if it bounds a 2-disk smoothly embedded into 4-space. We demonstrate a method to generate amphichiral knots using braid representations. Using a computer search with SnapPy, we find amphichiral knots with determinant 1 that are not slice. These knots, to the best of our knowledge, are the first known with these properties and give rise to order 2 elements in the *smooth integral homology cobordism group*, a group of intense interest to 4D topologists. We also discuss different kinds of *rotant operations*, which generalize mutations and involve cutting out a piece of a knot and rotating it 180°, assuming that the result is a knot. Manolescu and Piccirillo find two pairs of knots with the same Alexander and Jones polynomials, so that if any of them are slice, then 4D smooth Poincare conjecture is false. In this talk, we discuss rotant operations that relate their pairs of knots. We investigate in what contexts rotant operations preserve certain knot invariants, especially the Alexander and Jones polynomials, using skein relations. **[EA06163423]**

[Joint with Dr. Melissa Zhang]

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Properties and Parameters of Codes from Unit Graphs of Z_n

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Abstract of Report Talk: The unit graph $G(Z_n)$ of Z_n has the vertex set $V = \{0, 1, \ldots, n-1\}$, all of the elements of Z_n, and two distinct vertices $x, y \in Z_n$ are adjacent whenever $x + y$ is a unit in Z_n. We study binary codes $C(Z_n)$ obtained from the row span of adjacency matrices of $G(Z_n)$. We determine parameters of $C(Z_n)$ codes by grouping them according to the prime factorization of n and the gap size of $G(Z_n)$, where the gap size of $G(Z_n)$ is defined by $n/2 - \phi(n)$ and $\phi(n)$ is the Euler-phi function. We find classes of self-orthogonal and linear complementary dual (LCD) codes among $C(Z_n)$ codes and classify them according to the prime factorization of n and the gap size of $G(Z_n)$. **[EV04163744]**

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Weyl's Law on Compact Heisenberg Manifolds

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Abstract of Report Talk: In 1911, Hermann Weyl showed that the volume of a bounded domain Ω in $\mathbb{R}^n$ can be recovered from the spectrum of the Laplacian on Ω. Results of this form, now variously known as Weyl's law, have since been established for compact Riemannian manifolds and the Hodge Laplacian on differential forms. In other words, in Mark Kac's terminology, we can hear the volume of a Riemannian manifold via the Hodge Laplacian.

Is there a similar result for CR manifolds and the Kohn Laplacian? Developments for Weyl's law for the Kohn Laplacian on CR manifolds began to be considered in 1984 by Stanton and Tartakoff. It was shown that for certain embedded CR manifolds of hypersurface type, an analog of Weyl's law holds on forms but not functions. The case for functions remains open.

In this project, we present a similar result for both forms and functions on a family of CR manifolds that are not of hypersurface type. More precisely, let M be a compact quotient of the d-dimensional Heisenberg group by a lattice subgroup. We prove that the eigenvalue counting function $N(\lambda)$ for any fixed element of a family of second order differential operators $\{\mathcal{L}_\alpha\}$ on M has asymptotic behavior

$$\lim_{\lambda \to \infty} \frac{N(\lambda)}{\lambda^{d+1}} = C_{d,\alpha} \operatorname{vol}(M)$$

where $C_{d,\alpha}$ is a constant that only depends on the dimension d and the parameter α. As a consequence, we obtain Weyl's law for the Kohn Laplacian on M. Our main tools are Folland's description of the spectrum of $\mathcal{L}_\alpha$ and Karamata's Tauberian theorem. **[FC05234101]**

[Joint with Elena Kim, Yunus E. Zeytuncu]
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Number of regions created by random chords in circle

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Abstract of Report Talk: To continue the topic about random chords in the circle in the book "Geometric Probability", we will discuss the distribution of the number of regions created by a certain amount of random chords (n random chords) in the circle. We find that it converges to the normal distribution as n goes to infinity and the error can be bounded by Stein's method for Central Limit Theorem. **[FS26144557]**

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Bounds on Higher Gonalities of Graphs

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Abstract of Report Talk: Chip-firing games on graphs provide a graph-theoretic analogue to divisor theory on algebraic curves and, through the process of tropicalization, give a combinatorial method of studying algebro-geometric objects. Of particular interest is the r^{th} gonality of a graph, the minimum degree of a rank r divisor, which gives information about maps from curves to r-dimensional space. Much of the existing literature has focused on the first gonality $\text{gon}_1(G)$ of a graph, leaving many open questions for the more challenging higher gonalities, $\text{gon}_r(G)$ with $r \geq 2$. Our main results are the strongest known upper and lower bounds on $\text{gon}_r(G)$.

We prove that $\text{gon}_r(G)$ can be upper bounded using the r^{th} independence number of a graph G given assumptions on girth and minimum valence, generalizing a result for first gonality by Harper et al. in 2016. Our techniques for this include a construction of a winning divisor satisfying Dhar's criterion. We leverage this finding to prove an NP-hardness result for second gonality when restricted to multiplicity-free divisors. We also generalize the newly defined scramble number of a graph, a lower bound on first gonality, to r^{th} scramble number which provides a new lower bound on the r^{th} gonality of the graph. **[FL05154429]**

[Joint with Lisa Cenek, Liz Ostermeyer, Eyobel Gebre, Jason Meintjes, Shefali Ramakrishna] Received: August 20, 2021

Extending Erdos Distinct Distance Problems to Angles

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()
Williams College [Mentor:Charles Wolf]

Abstract of Report Talk: Paul Erdős famously said "[m]y most striking contribution to geometry is, no doubt, my problem on the number of distinct distances." That is, given n points in the plane, what is the minimum number of distinct distances among them? Erdős' initial asymptotic lower bound was $\sqrt{n}$ and he gave the integer lattice as an example of a point configuration with on the order of $n/\sqrt{\log n}$ distances. In 2010, Guth and Katz proved an asymptotic lower bound of $n/\log n$, finally answering Erdős' question in the plane for all practical purposes. Higher dimensional analogues of this question, while not fully resolved, have still been thoroughly investigated.

As the problem of distances has now been well-studied, we pivot to the problem of the minimum number of distinct angles among n points in $\mathbb{R}^d$, $A^d(n)$. For each of the following questions, we establish the first such bounds.

Using many circles in split dimensions (a modified Lenz construction), we show

$$A^d(n) \ \leq \ 2\left\lceil \frac{n}{\lfloor d/2 \rfloor} \right\rceil - 2.$$

In the plane, we consider several different restricted versions of this problem. Define $A_{\text{no3l}}(n)$ as the analogue over planar point sets and $A_{\text{gen}}(n)$ as the analogue over sets with no 3 collinear points and no four cocircular points (general position). Using regular polygons and recent progress on the Weak Dirac Conjecture we show

$$\frac{n}{6} \ \leq \ A_{\text{no3l}}(n) \ \leq \ n-2.$$

Letting $\lesssim$ denote asymptotic inequality, we prove that

$$n \ \lesssim \ A_{\text{gen}}(n) \ \lesssim \ n^{\log_2(7)}.$$

The lower bound follows from the bound for $A_{\text{no3l}}(n)$. We achieve the upper bound using a combinatorial argument bounding the number of translation equivalent triangles in hypercubes projected onto a generic plane.

We also consider a variant of the distinct angles problem. Letting $R(\mathcal{P})$ be the size of the largest subset of $\mathcal{P} \subset \mathbb{R}^2$ without repeated angles, we define

$$R(n) \ := \ \min_{|\mathcal{P}|=n} R(\mathcal{P}),$$

whose minimum is over all such $\mathcal{P}$ in general position. We prove that

$$n^{1/5} \ \lesssim \ R(n) \ \lesssim \ n^{\log_2(7)/3}.$$

The lower bound follows by bounding the number of pairs of angles with a given number of overlapping points and a probabilistic argument involving arbitrary point configurations. The upper bound follows from our construction for $A_{\text{gen}}(n)$. **[FH03142042]**

[Joint with Faye Jackson, Steven J. Miller, Eyvindur A. Palsson, Ethan Pesikoff]

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Phase Retrieval from Local Measurements via Lifting and Eigenvector-based Angular Synchronization

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Abstract of Report Talk: We study a computational approach to phase retrieval with application to ptychographic imaging. Phase retrieval is a challenging nonlinear inverse problem involving the recovery of a complex vector (sample), up to a single unimodular constant, from magnitude-only measurements. Ptychography refers to a computational imaging method where a sample of interest is recovered from a series of overlapping local magnitude-only measurements. We present herein a reconstruction algorithm which intakes ptychographic measurements and outputs an estimate of the sample. Pre-computation converts the nonlinear system inherent in our measurement model into a higher-dimensional linear system via structured lifting. The solution to this linear system can then be represented by a matrix with a cascading block structure. Our algorithm performs eigenvector-based angular synchronization on the solution to this linear system, recovering an estimate for the sample in question. We present numerical results concerning the noise robustness and computational efficiency of said algorithm. **[FJ31145140]**

[Joint with Yulia Hristova, Nicole Baker] Received: August 18, 2021

Consequences of the Packing Problem

Caleb Fong (cjxf@st-andrews.ac.uk)
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Abstract of Report Talk: In commutative algebra, there are various notions of taking a power of a monomial ideal – namely the ordinary power, the symbolic power, and the rational/real power. It is also possible to reframe the study of these powers into questions on polyhedral geometry. These notions and the relationships between them are a topic of active research. In this talk, we will present some novel results concerning some algebraic invariants of these monomial ideals. In particular, we will present some interesting consequences of the packing problem, a long-standing conjecture originally from the field of combinatorial optimization. **[FC05174408]**

[Joint with H. Bodas, B. Drabkin, S. Jin, J. Kim, W. Li, A. Seceleanu, T. Tang, B. Williams] Received: August 18, 2021

Infectious Disease Model- COVID 19- Volterra Integral Equation

Kate E Gilbert (kgilbert721@g.rwu.edu)
Roger Williams University [Mentor:Yajni Warnapala]

Abstract of Report Talk: Inspired by the COVID-19 pandemic, this research investigates the feasibility of obtaining good convergence results for a model of the nonhomogeneous Volterra integral equation of the second kind over the surface (geographic location). The Galerkin Method was used to numerically approximate the integral using Gaussian quadrature nodes. Volterra Integral Equations are widely used to model infection and recovery of disease in a population. This model accounts for the number of initially infected individuals, susceptible individuals, removed individuals, number of contacts per person, the recovery rate, age, the total population, and an unknown function that is hypothesized to be a variable of combining age, preexisting health conditions, income, access to healthcare, etcetera. This model specifically looks at COVID-19 in Brazil and South Africa for the first 300 days of the pandemic. The numerical results of this research are expected to find good convergence for this model as well as limitations of the model. The model is built on the assumption that the infection curves are characteristic of cubic and inverse tangent functions. The pandemic has been shown to have two waves of increased infections, both with similar cubic and inverse tangent patterns. The ISR model was applied to both waves of the pandemic with modifications. [GK06133312]

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Square Factors in Projective Character Degrees

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Texas State University [Mentor:Yong Yang]

Abstract of Report Talk: While the relationship between the irreducible character degrees of a group and the structure of the group itself has been studied extensively, a relatively small number of attempts have been made to extend these results to the more general projective characters. We progress towards generalizing such a result regarding the structure of a finite group whose character degrees are all square-free. We show that a minimal-order exception to our proposed result would be an extension of a finite simple group by a solvable group, and outline progress toward showing that such an exception would be simple. We also show that a finite simple exception would have to be of Lie type. We use a weighted pigeonhole argument using Deligne-Lusztig theory to handle some cases, and express how it may be used to solve others. We also discuss the merit of considering representations such as the Weyl representation. [GM04182040]

[Joint with Matthew Kortje, Megan Laurence, Zili Wang]

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Modeling Perceptual Machinery and its Effects on Dispersal

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Abstract of Report Talk: Dispersal is the phenomenon of an organism immigrating from the habitat of its birth to a new habitat. Though much progress has been made in modeling plant dispersal using lattice models, as shown in the work by Sato and Iwasa [SI], comparitively little has been done using lattices to model the dispersal of animals. As in [SI], we employ a lattice-logistic model and pair approximation to study changing population densities in an isotropic lattice. Unlike their model, this lattice is not homogeneous, and is occupied by organisms capable of perceiving differences in their enviroment. We find that our model shows qualitatively similar behavior to simulation results and that overall propagation is highest when organisms have no perceptual machinery. We proceed to develop the model further by introducing higher death rates in unfavorable lattice patches, which heavily impacts the densities at which maximal propagation occurs. Additionally, we discuss an implicit competition cost inherent in populations with a high degree of perceptual ability. **[GI03092326]**

[Joint with Skyler Moomey and Quinn Stoddard-O'Neill] Received: August 18, 2021

Generalized Two-player Pebbling Games on Simple Graphs

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Moravian University [Mentor:Eugene Fiorini]

Abstract of Report Talk: Graph pebbling is a mathematical game played on an undirected graph with no loops or multiple edges. An (a, b) graph pebbling game involves two players who alternate making moves in the following manner: removing a pebbles from a vertex and adding b pebbles to an adjacent vertex, with $a > b$. The first player having no available move loses the game. We determine some conditions for which the $(k + 1, k)$ graph pebbling game played on the complete graph K_n results in a first-player winning game or a second-player winning game. We also analyze the standard $(2, 1)$ graph pebbling game on other simple graphs. **[GM29131613]**

[Joint with Mia DeStefano, Joe Miller, Jacob Roeder, Wing Hong Tony Wong] Received: August 17, 2021

Braid Indices of 1-Bridge Braids

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Abstract of Report Talk: Many interesting 3-dimensional spaces arise as the complement of knots embedded into three-dimensional spaces. Thus, knots are an important tool in studying low-dimensional topology, the study of shapes and spaces of dimension at most four. An important and well-studied family of knots to consider are 1-bridge braids. To help us better understand these knots, we study a knot invariant called the braid index. Knot invariants are a crucial concept in knot theory because they can sometimes tell us when two knots are different. Employing Markov's Theorem and a result of Morton and Franks-Williams, we compute the braid index for any 1-bridge braid, K(w,b,t), in terms of the number of strands w, the bridge number b, and the number of twists t. Future work will look at the insights this yields about certain 3-dimensional manifolds. **[GD05175928]**

[Joint with Siddhi Krishna, Izah Tahir, Len White, Viridiana Neri] Received: August 16, 2021

Fixed point linking for self-mappings of the disk

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Abstract of Report Talk: We investigate the behavior of the fixed points of a homeomorphism H on the 2-dimensional disk admitting a periodic orbit. In the mapping torus for H, the fixed points of H and the periodic orbit each form closed braids. We can define the linking number of a fixed point Q as the linking number of its associated braid with that of the periodic orbit. We developed a Python program which groups the fixed points of H into Nielsen equivalence classes and calculates the linking difference of two distinct fixed point classes. To gain an understanding of the behavior of these fixed points, we then simulated 42,500 random maps of period five belonging to a class of maps (not necessarily homeomorphisms), for which the linking difference is well defined. Firstly, we recorded how often members of this class of maps were homeomorphisms. Secondly, since the set of linking differences of a map of period P is defined on $\mathbb{Z}_P$, we tracked how many of the P possible members of this set occur for a given map. Thirdly, we tracked the number of fixed point equivalence classes belonging to each map as well as the frequency of Nielsen paths. **[GC05205925]**

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Generalization of the Excess Area and Its Geometric Interpretation

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Abstract of Report Talk: The image area of $\mathbb{D}$ under $z \cdot h$ exceeds the image area under h (holomorphic on $\mathbb{D}$). In his book, *Hermitian Analysis*, John D'Angelo precisely determined how the excess area on $\mathbb{D}$ generated by the multiplier, z, grows. After replacing z (in $z \cdot h$) with a finite Blaschke product, we identify precisely how such an excess area grows. Further, we derive an upper and lower bound for the excess area growth associated with a finite Blaschke product in terms of the excess area with z. Additionally, we obtain an interesting identity after replacing holomorphic functions with harmonic functions on $\mathbb{D}$ in the excess area formulation. Furthermore, we move this formulation onto domains conformal to $\mathbb{D}$. Inspired by D'Angelo's work, we employ Stokes' Theorem relating integration over $\mathbb{D}$ to integration on $b\mathbb{D}$ and utilize the relationship between the L^2-norm of h and the ℓ^2-norm of the Taylor coefficients of h. We also apply holomorphic changes of coordinates and use the properties of the Blaschke product and its factors, along with the solution of the Dirichlet problem for the disk, the Cauchy Integral Formula, Cauchy's Theorem, some elementary inequalities, and properties of the Poisson kernel. **[GS04185549]**

[Joint with Haley Bambico]

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Writing a Product of Prime Powers as a Sum of Recurrence Terms

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Abstract of Report Talk: We begin by extending the work of Pink and Ziegler (2018) to find all integer solutions to Diophantine equations such as

$$F_{n_1} + F_{n_2} + F_{n_3} = 3^{z_1},$$

where (F_n) is the Fibonacci sequence. First, we find an explicit upper-bound on solutions (n_1, n_2, n_3, z_1) using lower bounds for linear forms in logarithms. Then, we introduce the LLL algorithm to significantly reduce this upper-bound, which allows us to perform a computational search for all integer solutions. In addition, we demonstrate how these techniques generalize to an algorithm for finding all integer solutions to equations consisting of a finite sum of binary recurrence terms equaling a product of prime powers. Finally, we also discuss how we can remove the binary condition and extend our family of Diophantine equations to consider recurrence sequences of higher order. **[HB02153101]**

[Joint with Luisa Velasco]

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Zeros of Real Random Polynomials Spanned by Bergman Polynomials

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Abstract of Report Talk: Let $f_n(z) = \sum_{j=0}^{n} \eta_j p_j(z)$, where $\{\eta_j\}$ are real-valued independent and identically distributed standard normal random variables, and $\{p_j(z)\}$ are Bergman polynomials on the unit disk of the form $p_j(z) = \sqrt{(j+1)/\pi}z^j$, $j \in \{0, 1, \ldots, n\}$. From well-known formulas for the expected number of real zeros and purely complex zeros of random polynomials, we prove that the expected number of zeros of $f_n(z)$ in the unit disk is asymptotic to $2n/3$, and of these zeros, asymptotically $\sqrt{2}\log n/\pi$ of them are on $[-1, 1]$. **[HD19110938]**

[Joint with Andrea Olvera and Jose Cruz-Ramirez]

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Combinatorics of Sol Lewitt's Three-Part Variations on Three Different Kinds of Cubes

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Abstract of Report Talk: In his art piece *Three-Part Variations on Three Different Kinds of Cubes*, Sol LeWitt poses a combinatorial question. Consider 3 different kinds of cubes: one with no open faces, one with one open face, and one with two opposing open faces. Taking three such cubes and stacking them vertically so that the corners of the touching faces are lined up and none of the open faces are facing up or down results in a stack of height 3. The question posed by the art piece is how many such stacks of height 3 exist that are unique under 3D rotation. We find an equation for the number of unique stacks of height n and as a consequence find that LeWitt missed one stack of height 3 in his art piece. We then find the composition of the missing stack computationally. **[HJ06165546]**

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Decomposability and Simon's Conjecture

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Abstract of Report Talk: A pure simplicial complex is said to be 'shellable' if there exists an ordering of its facets with particular intersection criteria. A shellable d-dimensional complex is 'shelling completable' if it can be extended to a shelling of the d-skeleton of a simplex. Simon's conjecture states that any shellable complex is shelling completable, and work from a previous REU established this for the special case of vertex-decomposable (VD) complexes.

To describe combinatorial obstructions of 'stuck' shellings, we prove that certain classes of shellable complexes can always be extended by a predetermined number of facets. We also show that extending shellings of VD subcomplexes to arbitrary VD complexes is not always possible. Generalizing flag simplicial complexes, we also examine the role that missing faces play in completions of shellings. **[HF06145505]**

[Joint with RJ Barnes, Anton Dotchermann, Cicely Henderson, Fran Herr, Suho Oh, Ethan Partida] Received: August 18, 2021

the Pop operator on m-Tamari Lattices

Letong Hong (clhong@mit.edu)
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Abstract of Report Talk: Motivated by the pop-stack-sorting map on the symmetric groups, Defant defined an operator $\mathsf{Pop}_M : M \to M$ for each complete meet-semilattice M by

$$\mathsf{Pop}_M(x) = \bigwedge(\{y \in M : y \lessdot x\} \cup \{x\}).$$

We study the dynamics of $\mathsf{Pop}_{\mathrm{Tam}_n(m)}$, where $\mathrm{Tam}_n(m)$ is the nth m-Tamari lattice. An element x is t-Pop-sortable if $\mathsf{Pop}^t(x) = \hat{0}$ the minimal element of the lattice. Let $h_t(m,n)$ denotes the number of t-Pop-sortable elements in $\mathrm{Tam}_n(m)$. We answer a conjecture by Defant that the generating function

$$\sum_{n\geq 1} h_t(m,n)z^n$$

is rational. We also enumerate the number of 3-Pop-sortable elements. Moreover, in the case of the Tamari lattice of Dyck paths (i.e. $m = 1$), we give explicit form of the generating function. **[HL06142056]**

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On the Two-Dimensional Jacobian Conjecture: The Generic Case

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Abstract of Report Talk: The (two-dimensional) Jacobian conjecture says that if the Jacobian of a polynomial map $(x, y) \mapsto (f, g)$ is a nonzero constant, then the map is invertible. The Newton polygon is a mapping of a polynomial to a convex hull in $\mathbb{R}^2$ based on the degrees of the variables in each term. By Abhynankar's reduction theorem, it is enough to consider polynomials whose Newton polygons have no edges with zero slopes. The first case to consider is the set of polynomials whose Newton polygons are quadrilaterals. In this case, we prove the conjecture when deg(f)/gcd(deg(f),deg(g)) = 2 and the leading forms of f and g are square-free. Most of the known results deal with the case when gcd(deg(f), deg(g)) is small. However, our approach allows the gcd to be arbitrary. Additionally, our approach can be extended to the case when deg(f)/gcd is arbitrary. Our idea is to use Magnus's formula, which is seemingly complicated; we are able to simplify this formula. [HW06130505]

[Joint with George Nasr, Li Li]

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Tick-Mouse Models for Lyme Disease with Seasonal Variation in Births, Deaths, and Tick Feeding

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Abstract of Report Talk: Lyme disease is the most prevalent vector-borne disease in the United States impacting the Northeast and Midwest at the highest rates and recently has been established in southeastern and south-central regions of Canada. Lyme disease is passed by the black-legged tick, *Ixodes scapularis*, infected with the *Borrelia burgdorferi* bacterium. Bacteria is secreted through the saliva of the tick when it bites a host. One of the hosts commonly fed on by *I. scapularis* is *Peromyscus leucopus* colloquially known as a white-footed mouse. Understanding this parasite-host interaction is critical because *P. leucopus* is one of the most competent reservoirs for Lyme disease. The cycle of infection is driven by larvae feeding on infected mice that molt into infected nymphs and then transmit the disease to another susceptible host such as a mouse or human. Lyme disease in humans is generally caused by the bite of an infected nymph. One of the main aims of this paper is to accurately model how seasonal variation in tick births, deaths, and feedings impact the infection cycle seen in the tick-mouse cycle. To account for the seasonal changes, we formulate a deterministic model using delay differential equations (DDE) with periodic parameters that depend on the spring, summer, fall, or winter seasons. Then we approximate this model using ordinary differential equations with multiple latent stages to represent the delays in molting or reproduction. Finally, we extend the ordinary differential equations (ODE) into a stochastic model, a continuous-time Markov chain (CTMC). The dynamics of the models are compared, ODE versus DDE and ODE versus CTMC. In addition, we calculate and discuss the relevance of the basic reproduction number, $\mathcal{R}_0$, for the mouse-tick cycle and determine the sensitivity of $\mathcal{R}_0$ with respect to changes in model parameters. [HK31174614]

[Joint with Johnny Rajala, Dr. Linda Allen, and Fahad Mostafa] Received: August 17, 2021

One Level Density of a Family of L-functions over Function Fields

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Abstract of Report Talk: Many problems in number theory can be reduced to understanding properties of zeros of related functions. For example, using tools from complex analysis (in particular contour integration of the logarithmic derivative of $\zeta(s)$, the Riemann zeta function) the distribution of zeros of $\zeta(s)$ implies the Prime Number Theorem: to first order, the number of primes up to x is approximately $x/\log x$; more refined knowledge of the spacings of the zeros leads to additional information about the primes. Further, the benefits of such a correspondence can be generalized, and for a variety of number theory problems one can associate a generalization of $\zeta(s)$, called an L-function, where we have similar connections. While many results in the field are conditional, conjecturally and in many special cases we are able to model the behavior in the number theory setting (the distribution of zeros) by that of eigenangles of matrices drawn from the classical compact groups. We often study these by random matrix theory (RMT).

In our case, we build upon previous work which has shown that L-functions associated with certain number fields' Hecke characters have zeroes which are modeled by the eigenvalues of symplectic matrices. We consider analogous L-functions associated with "super-even" characters in the function field setting. Though these characters have been studied from an RMT perspective as $q \to \infty$ (large finite fields), we instead consider the limit where the modulus of the Dirichlet character is large. We compute the limiting one-level density for this family of L-functions and we show that it matches a symplectic distribution for a class of test functions f whose Fourier transform $\hat{f}$ has compact support contained in $(-\alpha, \alpha)$ for $\alpha < 1$. We also calculate a lower-order term for the one-level density. The one-level density and its lower-order terms are useful for understanding the distribution of zeroes near the central point, where often interesting arithmetic happens; for example, the Birch and Swinnerton-Dyer conjecture (one of the Clay Millennial Problems) states that the order of vanishing of an elliptic curve L-function equals the rank of the group of rational points of the elliptic curve. **[IH05175741]**

[Joint with Dang Dang, Steven Miller, Ezra Waxman]

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The Bergman Game

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Abstract of Report Talk: Every integer $n > 0$ can be written uniquely as a sum of non-adjacent Fibonacci numbers $\{F_n\}$, called its Zeckendorf decomposition. For example $2021 = 1597 + 377 + 34 + 13$, which is $F_{15} + F_{12} + F_7 + F_5$ where we choose to set $F_0 = 1, F_1 = 2$. Earlier work analyzed a combinatorial game played on an infinite tuple of Fibonacci numbers; starting with n copies of F_1 players alternate by using moves arising from the Fibonacci recurrence, with the winner being the player who moves last and consequently forms the Zeckendorf decomposition of n. For $n > 2$, the second player to move has a winning strategy (though the proof is non-constructive), and all games take on the order of n moves.

Our work takes the Zeckendorf Game and removes the boundary conditions; this new game–the Bergman Game–is now played on a doubly infinite tape. Playing the Bergman game produces the unique base φ decomposition of n, where φ is the golden mean. For example:

$$2021 = \varphi^{-16} + \varphi^{-11} + \varphi^{-6} + \varphi^{-3} + \varphi + \varphi^5 + \varphi^{10} + \varphi^{13} + \varphi^{15}.$$

Previous research has extensively studied base β representations for any real number $\beta > 1$ via Ergodic theory and symbolic dynamics. Our research presents a new way of studying these base β representations whenever β satisfies the characteristic polynomial of a Positive Linear Recurrence Sequence (PLRS) with non-increasing coefficients using classes of games similar to the Zeckendorf Game.

The rules of the game for $\beta = \varphi$ are extremely simple. Two ones at adjacent indices may be combined to create a new one farther right, and a two can be split into two ones as below:

Combine move:	$(\dots, 1, 1, 0, \dots) \to (\dots, 0, 0, 1, \dots)$
Split move:	$(\dots, 0, 0, 2, 0, \dots) \to (\dots, 1, 0, 0, 1, \dots)$.

The last player to make a move wins. We call each one in the tuple a summand in the spirit of base φ decompositions.

Using invariants and monovariants, we prove a bound of $2n + 2\log_\varphi n + 2$ on the length of tape needed to play the game with n summands, as well as an upper bound of $O(n^2)$ on how long a game will take on any initial state and a tight lower bound of $\Omega(n)$ on how long the game takes from the initial state (n).

We also demonstrate that games following a strategy which prioritizes splits from the left and subsequently combines from the right require a tape of length $2n + O(\log n)$ and terminate in $n^2 + O(n \log n)$ moves. To prove the lower bound on tape length, we use the pigeonhole principle to show that this strategy requires a tape of length n, and then we iteratively improve this in order to achieve $2n + O(\log n)$. This game analysis shows that our above upper bounds are tight in the dominating terms.

We further generalize these results to a class of games mirroring the Generalized Zeckendorf Game, which gives rules for a Generalized Bergman Game modeled on any PLRS with non-increasing coefficients. We are able to prove comparable results on the left bound and the termination time for this more general class of games. Even more generally, we are able to show that a wide class of games which are summand conservative (i.e. no move increases the total number of summands) terminate in exponential time. **[JF04214650]**

[Joint with J. Dell, E. Pesikoff, I. Durmic, H. Fleischmann, Y. Yang, I. Mijares, A. Smith, B. Baily] Received: August 18, 2021

Coupled Solutions to a Discrete Nabla Caputo Fractional Boundary Value Problem

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Abstract of Report Talk: Discrete fractional calculus is a relatively new area of mathematics that seeks to develop discrete analogues for the continuous fractional calculus. The nabla Caputo fractional difference is defined in terms of a fractional integral and a whole-order nabla difference, where the fractional integral allows us to integrate by any positive real number order. Assume $f : \mathbb{N}_{a-N+1} \to \mathbb{R}$ and $\nu > 0$. Specifically, the ν-th order nabla Caputo fractional difference of f is defined by

$$\nabla^{\nu}_{a*} f(t) = \nabla_a^{-(N-\nu)} \nabla^N f(t), \text{ for } t \in \mathbb{N}_{a+1} \text{ and } N = \lceil \nu \rceil.$$

We consider the existence, uniqueness, and Hyers-Ulam stability of coupled solutions to the following nabla Caputo fractional boundary value problem:

$$\begin{cases} -\nabla^{\nu_1}_{a*} x_1(t) = f_1(t, x_1, x_2), & t \in \mathbb{N}^b_{a+1}, \\ -\nabla^{\nu_2}_{a*} x_2(t) = f_2(t, x_1, x_2), & t \in \mathbb{N}^b_{a+1}, \\ \nabla^i x_1(a-1) = \nabla^i x_2(a-1) = 0, & i \in \mathbb{N}_0^{N-2}, \\ \nabla^j x_1(b) = \nabla^j x_2(b) = 0, & \text{where } j = 0 \text{ or } j = 1. \end{cases}$$

Using the Banach Fixed Point Theorem we establish sufficient conditions for the nonlinear nonhomogeneous functions f_1 and f_2 that yield the existence and uniqueness of coupled solutions. Using a method from Urs (2013) we also prove the Hyers-Ulam stability of those solutions. Finally, we present an example of coupled solutions to a particular nonlinear system and detail a method for computing their numerical approximations using Picard iterates.

[JC06104145]

[Joint with Kevin Ahrendt] Received: August 18, 2021

Quantitative Helly-type theorems via sparse approximation

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Abstract of Report Talk: Helly's theorem is a fundamental theorem in convex geometry. Its finite version states that the intersection of a finite family of convex sets is empty if and only if there exists a subfamily of $d+1$ convex sets such that its intersection is empty. In 1982, Bárány, Katchalski and Pach introduced the following quantitative versions of Helly's theorem: there exist positive constants $c(d), v(d)$ such that for a finite family $\mathcal{F}$ of convex sets, one may select $2d$ members such that their intersection has diameter at most $c(d)\,\mathrm{diam}(\bigcap\mathcal{F})$, or has volume at most $v(d)\,\mathrm{vol}(\bigcap\mathcal{F})$. The problem of finding the optimal values of $c(d)$ and $v(d)$ has enjoyed special interest in recent years. Improving upon the ideas that Naszódi introduced in 2016, Brazitikos in 2017 found the current best bound for volume, $v(d) \leq (cd)^{3d/2}$ for a constant $c > 0$. For the diameter, Ivanov and Naszódi found the best known bound earlier this year, $c(d) \leq (2d)^3$.

We show that given a finite family $\mathcal{F}$ of convex sets, one can select at most $2d$ members such that their intersection sits inside a scaled version of $\bigcap\mathcal{F}$ for a suitable location of the origin. As an application, we obtain an improvement on the diameter bound, $c(d) \leq 2d^2$, and retrieve the best known bound for $v(d)$. The crux of the proof is a sparse approximation result for polytopes: given a convex polytope Q, we find a set of at most $2d$ vertices such that their convex hull Q' satisfies $Q \subseteq \lambda Q'$ for $\lambda = -2d^2$ and a suitable location of the origin. This polytope approximation result is also of independent interest. **[KM04092856]**

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Extending support in calculating the n-level density of low lying zeroes of families of L-functions

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Williams College [Mentor:Steven J. Miller]

Abstract of Report Talk: The distribution of zeros of L-functions is central to some of the most important problems in number theory. The prototypical L-function is the Riemann zeta function $\zeta(s) := \sum_{n=1}^{\infty} 1/n^s = \prod_{p \text{ prime}}(1 - 1/p^s)^{-1}$ for $\mathrm{Re}(s) > 1$. Standard techniques from complex analysis relate zeros of $\zeta(s)$ to the distribution of prime numbers. Zeros of other L-functions yield information about many important problems, such as the class number formula, primes in arithmetic progressions, and the odd Goldbach conjecture.

Montgomery and Dyson's work in 1970 showed a surprising connection between spacings of zeros of L-functions and eigenvalues of ensembles of random matrices. However, the quantities they studied are insensitive to finitely many zeros, and thus cannot determine behavior of the zeros near the central point $s = 1/2$. Studying these low-lying zeros is important to major open problems such as the Birch and Swinnerton-Dyer conjecture, which yields information about the structure of rational points on elliptic curves.

In 1999, Katz and Sarnak conjectured that the n-level density, a statistic on the distribution of spacings between low-lying zeroes of a family of L-functions, is the same as the distribution of eigenvalues of a family of orthogonal random matrices. The n-level density relies on an even Schwartz test function, and increasing the support of its Fourier transform provides better understanding of the zeros near the central point. Previous work has confirmed the Katz-Sarnak Density Conjecture for some families of L-functions but with significant restriction on this support.

In 2006, Hughes and Miller computed the n^{th} centered moments of the 1-level density of families of cuspidal newforms split by the sign of their functional equation for test functions of support $\left(-\frac{1}{n-1}, \frac{1}{n-1}\right)$, conditional on the Generalized Riemann Hypothesis (GRH). They also prove that their results agree with random matrix theory as predicted by the Katz-Sarnak Density Conjecture.

We extend much of Hughes' and Miller's work to support $\left(-\frac{1}{n-a}, \frac{1}{n-a}\right)$ for any positive integer $a \leq n/2$. Our results are contingent on GRH and on a certain sum over Dirichlet characters being sufficiently small, which is likely true due to the highly oscillatory nature of Dirichlet characters. Via the Petersson formula, the n^{th} centered moments can be written as a sum of terms involving Bessel-Kloosterman sums. For smaller support, many of these terms are negligible as the level goes to infinity. As we increase support, the non-negligible Bessel-Kloosterman terms become more complicated and more numerous, which makes increasing support difficult. By generalizing the results of Hughes and Miller, we determine exactly which terms become negligible over support $\left(-\frac{1}{n-a}, \frac{1}{n-a}\right)$, while combinatorially handling the terms that remain. Our work helps establish new evidence for the Katz-Sarnak Density Conjecture in larger support and aids in bounding the order of vanishing of L-functions at the central point.
[KS06144156]

[Joint with Justine Dell, Alicia Smith Reina, Alexander Shashkov, Yingzi Yang]

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Mathematical animation and fusion categories

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Abstract of Report Talk: Manim is a mathematical animation library developed by the popular YouTuber 3Blue1Brown. It lends itself nicely for math education and exposition in highly visual fields. The study of fusion categories, categories which can describe the merging and splitting of elementary particles, is such a field. In particular, the so-called graphical calculus is a modern technique for the classification and construction of fusion categories. Over the summer, I have helped develop videos that provide accessible introductions to fusion category theory using this library. In this talk, I will present examples of the power of Manim by showing animations which introduce the graphical calculus for fusion categories and continue by discussing the code behind the animations and opinions on mathematical exposition to give some intuition for how one could program clean and informative videos for other fields of mathematics using this library. **[KQ28120050]**

[Joint with Giovanni Ferrer] Received: August 18, 2021

Constructing fusion category representations with the Jellyfish algorithm

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Abstract of Report Talk: The graphical calculus is a modern technique for classifying and constructing "unitary fusion" categories, categories which model elementary particles that merge and split. In particular, if one can obtain enough diagrammatic relations to evaluate all "closed" diagrams, this graphical calculus uniquely describes the category. However, in order to prove existence, one must also find a representation of the category. Cuntz algebras, a type of C^*-algebra, are a natural home for the representations of many fusion categories. The Jellyfish algorithm allows us to use diagrams to find what these Cuntz algebra representations should be in an intuitive way. However, a full treatment of this translation is not in the literature. Using this computation, we can then prove existence by showing that this construction is indeed a well-defined representation in the associated Cuntz algebra. In this talk, I will apply this process on the unitary fusion category with Fibonacci fusion rules. **[KQ04134231]**

[Joint with Daniel Wallick and Quan Chen] Received: August 18, 2021

Characterizing Non-abelian Simple Groups with Codegrees

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Abstract of Report Talk: Let G be a finite group and $\chi \in \mathrm{Irr}(G)$. Then $\mathrm{cod}(\chi) = \frac{|G:\ker(\chi)|}{\chi(1)}$ and $\mathrm{cod}(G) = \{\mathrm{cod}(\chi) | \chi \in \mathrm{Irr}(G)\}$. Previous study of codegrees includes the relationship between element order and codegrees and classifying groups with few codegrees. Huppert's conjecture seeks to characterize finite non-abelian simple groups using the set of character degrees. We consider a similar conjecture for codegrees and show that the set of codegrees of M_{11} and $\mathrm{PSL}(3,3)$ determine the groups up to isomorphism. [KM05185046]

[Joint with Michael Gintz, Megan Laurence, Dr. Yang Liu, Zili Wang, Dr. Yong Yang] Received: August 17, 2021

On Morita Equivalence of Inverse Hulls of Markov Shifts

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Abstract of Report Talk: We study the relationship between stable isomorphism of Cuntz-Krieger C^*-algebras and "Morita equivalence" of shift inverse semigroups (inverse hulls). In the known C^*-algebra theory stable isomorphism corresponds to flow equivalence of shift spaces. In the construction of the C^*-algebra of a semigroup S, a quotient $S/\leftrightarrow$ is instrumental. In the case where S is the inverse hull of a Markov shift we show, under a mild condition, that $S/\leftrightarrow$ is isomorphic to the inverse hull of a flow equivalent Markov shift. In doing so we initiate the study of a weaker notion of Morita equivalence of inverse hulls that corresponds to stable isomorphism of their C^*-algebras. [KA23143018]

[Joint with David Milan] Received: August 16, 2021

Partial Differential Equations that Model Cancer Tumor Cell Growth

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Abstract of Report Talk: We study a system of partial differential equations modeling cancer tumor growth. The model is of free-boundary type, which means it describes the tumor as a subset of Euclidean space that evolves in time. The tumor's growth is inhibited by pressure inside the tissue. The model also takes into account friction between the tumor cells themselves. We studied the problem in one dimension, in which case it is possible to reduce it to a system of ODEs. Our main result concerns the long-time behavior of the model: we proved that the tumor's growth is linear in time. To do this, we first computed an explicit formula for solutions of two of the ODEs. Then we analyzed the ODE that determines the spread of the boundary of the tumor. In addition to establishing this result, we also investigated (numerically and via formal computations) the inviscid (frictionless) limit of the model. **[LR02174305]**

[Joint with Anthony Sulak] Received: August 17, 2021

Recognizing Twisted n-Eared Rabbit Polynomials

Lily Li (lilyli@berkeley.edu)
Georgia Institute of Technology [Mentor:Dan Margalit]

Abstract of Report Talk: From a topological point of view, a polynomial is a branched cover of the Riemann sphere over itself. In this talk, we seek to understand how to recognize whether branched covers arise from polynomials. The work builds upon Bartholdi-Nekrashevich and Belk-Lanier-Margalit-Winarski, both of which determined Thurston equivalence classes of the image of the Rabbit polynomial under powers of Dehn Twists around its ears. Using this topological viewpoint, we generalize these results to larger classes of topological polynomials such as the n-eared rabbit polynomials, and we extend the scope of the classification to include all elements of PMod in addition to powers of particular Dehn Twists. **[LL06232008]**

[Joint with Caleb Partin] Received: August 19, 2021

Visualization of a Brownian Motion on Hopf Fibration

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Abstract of Report Talk: In this project, the main goal is visualizing a Brownian Motion on Hopf fibration. We first visualize the Hopf fibration using the modified stereographic projection. The Hopf fibration is a mapping from $S^3 \to S^2$. For each point in S^2, the fiber is the pre-image. Then we visualize a Brownian motion beginning by a Brownian motion on $\mathbb{CP}^1$, then changing the coordinate to construct a path on Hopf fibration. **[LZ06065143]**

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Counting the Number of Permutations on $[n]$ without Strong Fixed Points and Small Descents

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Moravian University [Mentor:Eugene Fiorini]

Abstract of Report Talk: A **strong fixed point** in a permutation π occurs when $\pi(k)$=k, $\pi(i) < k$ for $i < k$ and $\pi(j) > k$ for $j > k$. A **small descent** in a permutation π is any pair $\pi(j), \pi(j+1)$ in the permutation where $\pi(j+1) = i$ and $\pi(j) = i+1$ for some $1 \leq i, j \leq n$. In this presentation, we count the number of permutations on $[n]$ containing no strong fixed points and no small descents via inclusion-exclusion. Our results are motivated by a problem concerning assignment graphs of prescribed girth. **[LX23140837]**

[Joint with Jared Glassband, Garrison Koch, and Evan Sabini]

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The Artin-Hardy-Littlewood Conjecture

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Abstract of Report Talk: For a fixed integer g, we say a prime number p is an *Artin Prime for the root g* if g mod p generates the cyclic group $(\mathbb{Z}/p\mathbb{Z})^*$. Just as for ordinary primes, the asymptotic distribution of Artin Primes and some related properties are quite mysterious and of great mathematical interest. While many results are known under the Generalised Riemann Hypothesis, a simple heuristic model by Moree gives the same answers on many occasions, such as the count of Artin Primes in Arithmetic Progressions.

We extend Moree's Model to obtain a conjecture on the asymptotic number of Artin Prime Tuples of fixed spacing, i.e., Artin Primes of the form $(p + d_1, p + d_2, \ldots, p + d_k)$. This generalises a previous conjecture on Artin Twin Primes (the case $k = 2$), using a generalised Hardy-Littlewood Conjecture on ordinary prime tuples and manipulation techniques from analytic number theory. We also report on large scale numerics that support the accuracy of this conjecture. These results suggest that the distribution of Artin prime tuples, amongst the ordinary prime tuples, is largely governed by a Poisson binomial distribution. **[LM05152345]**

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Probabilistic Chip-Collecting Games with Various Winning Conditions

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Moravian University [Mentor:Joshua Harrington]

Abstract of Report Talk: Let $a < b < \min\{n_1, n_2\}$. In a two player random race game, Alice flips a fair coin. If she gets heads, Alice gets a chips and Bob gets b chips. Otherwise, Alice gets b chips and Bob gets a chips. We examine two types of random races, referred to as modular and non-modular games. In a modular game, Alice wins if she obtains a number of chips congruent to 0 modulo n_1. Bob wins if he obtains a number of chips congruent to 0 modulo n_2 before Alice wins. In the non-modular game, Alice wins if she gets n_1 or more chips, whereas Bob wins if he gets n_2 or more chips before Alice wins. Previous studies considered the case when $n_1 = n_2$.

In the modular game, we address the worst-case scenarios in terms of Bob's winning probability. In the non-modular game, we fully determine the winning probabilities of each player. In the process, we prove several conjectures postulated in previous studies. **[LX30121259]**

[Joint with Xuwen Hua, Xufei Liu, Alex Nash, Rodrigo Rios, Tony W. H. Wong] Received: August 17, 2021

Distinct distances between two parabolas in 3D

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City University of New York [Mentor:Adam Sheffer]

Abstract of Report Talk: Distinct distances between points on curves in $\mathbb{R}^2$ have been widely studied. However, much less is know about the three-dimensional case. Mathialagan and Sheffer recently studied the problem of distinct distances between points on circles in $\mathbb{R}^3$. We extend their analysis to study distinct distances between points on parabolas. For point sets $\mathcal{P}_1$ and $\mathcal{P}_2$ in $\mathbb{R}^3$, let $D(\mathcal{P}_1, \mathcal{P}_2)$ be the number of distinct distances spanned by pairs from $\mathcal{P}_1 \times \mathcal{P}_2$. We prove the following theorem:

Theorem 1. *Let B_1 and B_2 be two parabolas in $\mathbb{R}^3$.*

(a) Suppose that after rotation and translation B_1 and B_2 can be parametrized by $\gamma_1(s) = \{(s, as^2, 0) : s \in \mathbb{R}\}$ and $\gamma_2(t) = \{(0, q - at^2, t) : t \in \mathbb{R}\}$ for some $a > 0$ and $q \in \mathbb{R}$. Then, there exist sets $\mathcal{P}_1 \subset B_1$ of m points and $\mathcal{P}_2 \subset B_2$ of n points such that

$$D(\mathcal{P}_1, \mathcal{P}_2) = \Theta(m + n).$$

(b) Suppose that B_1 and B_2 cannot be parametrized as in (a). Then, for all sets $\mathcal{P}_1 \subset B_1$ of m points and $\mathcal{P}_2 \subset B_2$ of n points we have that

$$D(\mathcal{P}_1, \mathcal{P}_2) = \Omega\left(\min\left\{m^{2/3}n^{2/3}, m^2, n^2\right\}\right).$$

[LJ05234528]

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Natural Search Algorithms Modeled by Biased Random Walks

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Washington State University [Mentor:Nikolaos Voulgarakis]

Abstract of Report Talk: The random walk is a fundamental process in many aspects of nature, from the way our immune system searches for and eliminates antigens, to a super predator hunting for food. However, up until this point, there are no models that accurately describe naturally occurring processes in which a walker is surrounded by a homogeneous field of targets. This project involves using computational and analytical methods from probability theory and stochastic differential equations in order to formulate a new mathematical model, which can then be parameterized by directly comparing numerical predictions with pre-existing experimental data. Through running thousands of simulations using WSU's high-performance computing cluster, we have found that a walker will move along a self-generated gradient of information, and will undergo various phase transitions as parameters such as density and velocity are changed. These results are essential to better understanding biological processes like cell motility, which are in turn vital for advancing drug therapies and developing policies for public health and safety. **[LA05052232]**

[Joint with Nikolaos Voulgarakis] Received: August 17, 2021

Three-layer multi-criteria approach to assessing risk of contracting COVID-19 for various activities

Edwin Lu (edwinlu99@gmail.com)
College of William and Mary [Mentor:Gexin Yu]

Abstract of Report Talk: A three-layer multi-criteria Analytical Hierarchy Process (AHP) was used to create a model that, given a user's local infection data, how often they leave their home, and other information, could estimate the likelihood of them contracting COVID 19. Factors such as workplace sanitation, travel frequency, and social distancing were considered. Several pairwise comparison matrices were constructed based on the safety of each factor, and the relative importance of each factor can be calculated by finding the maximum eigenvalue and its corresponding eigenvector. The weight of each factor can be applied based on individual behavior and lifestyles. The calculated values reflect the risk the individual currently faces on a daily basis. Additionally, what-if scenarios can be simulated to give individuals a better understanding of their potential exposure if they change certain aspects of their daily routine. Future research can be developed by conducting surveys to define weights for different populations. **[LE29215903]**

[Joint with N/A] Received: August 16, 2021

How Best to Count Smooth Numbers: A Problem in Computational Number Theory

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Abstract of Report Talk: Let $x, y \in \mathbb{Z}$ such that $x \geq y > 0$. We say x is y-smooth if for every prime divisor p of x, we have $p \leq y$. Our work examines $\Psi(x, y)$, which denotes the number of y-smooth integers less than x. Numerous applications require the computation of $\Psi(x, y)$. Examples include optimizing the running time of integer factoring algorithms and the post-quantum key exchange protocol proposed by Couveignes, Rostovtsev, and Stolbunov, which requires that smooth numbers be generated uniformly at random with known prime factorization. Thus, it is relevant to examine algorithms for computing and estimating $\Psi(x, y)$.

We can compute $\Psi(x, y)$ exactly with Buchstab's identity or approximate it with reasonable accuracy using methods based on a theorem from Hildebrand and Tenenbaum. However, other algorithms offer advantages depending on the values of x and y. First, we examine the fastest approximation, $\Psi(x, y) \approx x \cdot \rho(u)$, where $u = \log x / \log y$ and $\rho(u)$ is the Dickman function. This method is only valid for y greater than the cutoff, $L(x)$. Assuming the extended Riemann hypothesis, Hildebrand gives $L(x) = (\log x)^{2+\varepsilon}$. Comparing the Dickman ρ method to the Hildebrand-Tenenbaum estimates, we propose a value of ε to generate the cutoff $L(x)$ in practice. Then, we present a new algorithm based on a theorem from Ennola, which is proven to produce a valid estimate for $2 \leq y \leq (\log x)^{\theta}$ with $0 < \theta < 3/4$. By comparing estimates generated by this algorithm to Buchstab's identity and the Hildebrand-Tenenbaum estimates, we show that the algorithm performs far better than what is guaranteed by Ennola's theorem. With these results, we propose rough guidelines for choosing an algorithm given values for x and y. **[MC01132140]**

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Generalizations of the Yao-Yao partition theorem and the central transversal theorem

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Abstract of Report Talk: We generalize the Yao–Yao partition theorem by showing that for any smooth measure in $\mathbb{R}^d$ there exist equipartitions using $(t + 1)2^{d-1}$ convex regions such that every hyperplane misses the interior of at least t regions. In addition, we present tight bounds on the smallest number of hyperplanes whose union contains the boundary of an equipartition of a measure into n regions. We also present a simple proof of a Borsuk–Ulam type theorem for Stiefel manifolds that allows us to generalize the central transversal theorem and prove results bridging the Yao–Yao partition theorem and the central transversal theorem. **[MM23092041]**

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On good infinite families of toric codes or the lack thereof

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Abstract of Report Talk: A toric code, introduced by Hansen to extend the Reed-Solomon code as a k-dimensional subspace of $\mathbb{F}_q^n$, is determined by a toric variety or its associated integral convex polytope $P \subseteq [0, q-2]^n$, where $k = |P \cap \mathbb{Z}^n|$ (the number of integer lattice points of P). There are two relevant parameters that determine the quality of a code: the information rate, which measures how much information is contained in a single bit of each codeword; and the relative minimum distance, which measures how many errors can be corrected relative to how many bits each codeword has. Soprunov and Soprunova defined a good infinite family of codes to be a sequence of codes of unbounded polytope dimension such that neither the corresponding information rates nor relative minimum distances go to 0 in the limit. We examine different ways of constructing families of codes by considering polytope operations such as the join and direct sum. In doing so, we give conditions under which no good family can exist and strong evidence that there is no such good family of codes. **[MR28122241]**

[Joint with Mallory Dolorfino]

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Recurrence for the Threshold Frog Model

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CUNY Baruch College [Mentor:Matthew Junge]

Abstract of Report Talk: Frog models are random two-type particle systems that feature rapid activation. They are used as toy models to understand basic mechanisms in epidemics, rumor spread, combustion, and nuclear fission. We study the frog model on infinite trees with initially an independent Poisson-m-distributed number of sleeping frogs at each site. Awake frogs jump at random between adjacent vertices and wake any sleeping frogs they visit. The main question of interest, which went unsolved for over a decade, was whether or not the model is recurrent i.e., whether or not every frog eventually wakes up. We answer a generalization of this question for the threshold variant in which sleeping frogs require multiple visits, rather than a single visit, in order to wake up. Using a new technique, we prove that the threshold frog model can be made recurrent so long as m is large enough. **[MZ08145114]**

[Joint with Matthew Junge, Tobias Johnson, Lily Reeves]

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Bounding Gonality of Circulant Graphs

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Shefali Ramakrishna
Williams College [Mentor:Ralph Morrison]

Abstract of Report Talk: Stemming from divisor theory on algebraic curves, the Baker-Norine chip-firing game has developed a rich theory in its own right. Algebro-geometrically defined as the minimum degree of a rational map from a curve to the projective line, the graph theoretic analog of *gonality* has provided a new avenue for proving powerful results in algebraic geometry.

We study the gonality of circulant graphs, a broad family encompassing a wide variety of well known families including complete and Harary graphs. We provide a constructive proof that $\text{gon}(\text{Ci}_n(J)) \leq 2\sum_\alpha j_\alpha^2$ for integers j_α in fixed adjacency list J, an upper bound which holds for any n, a rare result for a general family of graphs. Our primary techniques include an algorithmic approach to constructing a positive rank divisor for all large n, based on adjacency list J.

Our results go deeper for the Harary graphs $H_{2k,n}$ where we prove that if a particular Harary graph ever hits the upper bound, so do all larger Harary graphs with the same adjacency list. Using computational tools developed by SMALL 2021, this allows us to fully characterize the gonality of Harary graphs $H_{4,n}$. [MJ05154331]

[Joint with Lisa Cenek, Lizzie Ferguson, Cassandra Marcussen, Liz Ostermeyer]

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Classifying 2-Dimensional Real Algebraically Defined Graphs by Diameter

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Moravian University [Mentor:Brian Kronenthal]

Abstract of Report Talk: A 2-dimensional algebraically defined graph $\Gamma_{\mathcal{R}}(f(X,Y))$ is a bipartite graph, constructed using a ring $\mathcal{R}$ and a bivariate function f, where each partite set is a copy of $\mathcal{R}^2$. In this graph, two vertices (a_1, a_2) and (x_1, x_2) are adjacent if and only if $a_2 + x_2 = f(a_1, x_1)$. The study of algebraically defined graphs can be motivated by incidence geometry, as every graph $\Gamma_{\mathbb{F}_q}(f)$ with girth 6 can be completed to a projective plane of order q.

Previously, all girth 6 graphs $\Gamma_{\mathbb{C}}(f)$ were proved to be isomorphic; however, it was unknown whether all girth 6 graphs $\Gamma_{\mathbb{R}}(f)$ were isomorphic. To address this question, we first prove that whenever p is a polynomial, $\Gamma_{\mathbb{R}}(p)$ has diameter 4 or 5. Moreover, we classify infinite families of such graphs by diameter, including a proof that all known girth 6 graphs $\Gamma_{\mathbb{R}}(p)$ have diameter 4. Ultimately, we use these tools to prove that the two girth 6 graphs $\Gamma_{\mathbb{R}}(XY)$ and $\Gamma_{\mathbb{R}}(X^3Y^3 + XY)$ are non-isomorphic. **[MJ30153236]**

[Joint with Jacob Roeder, Hani Samamah, Wing Hong Tony Wong] Received: August 17, 2021

New Bounds on Sunflower Variants

Anushka Murthy (am5085@columbia.edu)
Baruch College, CUNY [Mentor:Guy Moshkovitz]

Abstract of Report Talk: We study variants on the Sunflower Conjecture posed by Erdös and Szemerédi. In particular, we say that a family of r binary vectors of length n is an "*(r,Q)-system*" if the number of 1's in every column is an element of Q when the family is viewed as a $r \times n$ binary matrix. We then define an "*(r,Q)-free*" family to be a family of binary vectors such that no r vectors form an (r,Q)-system, and ask what bounds exist on the maximum size of an (r,Q)-free family; the original Sunflower Conjecture problem is the special case where $Q = \{0, 1, r\}$. For $Q = \{0, d, d+1, \ldots, r\}$, we establish nearly tight upper and lower bounds on this maximum size. Our bounds generalize results found by Alon, Fachini, and Körner on "r-thin" families, which are a special case of our result with $d = 2$. We also study a "focal" version of our sunflower variants, which additionally requires that the sunflower has a vector x such that at least d of the other $r-1$ vectors agree with x at each coordinate. We again establish nearly tight upper and lower bounds on the maximum size of a family with no focal variant; this generalizes a result of Alon and Holtzman, which focused on the specific case where $d = r-2$. At their core, our upper bound proofs rely on several clever applications of the generalized pigeonhole principle to find many collections of vectors with similar behavior. We take a probabilistic approach for our lower bounds, utilizing "random choice with alterations" method **[MA06141001]**

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Thresholds for the Existence of Similar Area Configurations

Arian Nadjimzadah (anadjimz@u.rochester.edu)
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Williams College [Mentor:Eyvindur Palsson]

Abstract of Report Talk: For a compact set $E \subset \mathbb{R}^d$, define its distance set

$$\Delta(E) = \{|x-y| : x, y \in E\}.$$

The Falconer distance conjecture, posed in 1985, states that $\Delta(E)$ has positive Lebesgue measure when the Hausdorff dimension of E (a finer notion of the size of subsets of $\mathbb{R}^d$) is greater than $d/2$. This problem has been a central enterprise in geometric measure theory, just as its discrete cousin, the Erdős distance conjecture, has been in discrete geometry. As in the discrete setting, many generalizations of the Falconer problem have been explored, both in the direction of choosing an operation other than distance and in considering more general configurations than a single "edge."

Belmiro Galo and Alex McDonald explored the analogous question in the case of determinants. Let $k \geq d \geq 2$ and $E \subset \mathbb{R}^d$ compact. Define

$$\mathcal{A}_{k,d}(E) = \left\{ (\det(x_{j_1}, \ldots, x_{j_d}))_{1 \leq j_1 \leq \cdots \leq j_d \leq k} : \{x_j\}_{j=1}^k \subset E \right\},$$

the $\binom{k}{d}$-area configuration of E. Galo and McDonald established values of $s_{k,d} < d$ so that if E has dimension greater than $s_{k,d}$ then $\mathcal{A}_{k,d}(E)$ has positive Lebesgue measure. Such rigid configurations often admit convenient group action frameworks. In this case, all k-tuples of points with the same area configuration are equivalent modulo $\mathrm{SL}_d(\mathbb{R})$.

Greenleaf, Iosevich, and Mkrtchyan recently initiated the study of *similar* configurations. They showed that for every $r \in (0, \infty)$, the collection of rigid distance configurations which are similar by scale factor r has positive Lebesgue measure. Here they crucially use the action of the Euclidean group.

We take inspiration from these papers to extend the discussion of similar configurations to areas in $\mathbb{R}^d$. In particular, define

$$\mathcal{A}^r_{k,d}(E) = \{t \in \mathcal{A}_{k,d}(E) : rt \in \mathcal{A}_{k,d}(E)\} \subset \mathcal{A}_{k,d}(E),$$

the similar $\binom{k}{d}$-area configuration. We show $\mathcal{A}^r_{k,d}(E)$ has positive Lebesgue measure when the dimension of E is greater than the Galo-McDonald threshold $s_{k,d}$. At the core of our argument, we take advantage of the Iwasawa decomposition of $\mathrm{SL}_d(\mathbb{R})$ to apply a change of variables.

We also initiate the study of similar area-tree configurations, which are at the other end of graph rigidity. Consider the analogous configuration set

$$\mathcal{A}_T(E) = \{(x_i \cdot x_j^{\perp})_{(i,j) \in \mathcal{E}(T)} : \{x_j\}_{j=1}^k \subset E\}.$$

We show that $\mathcal{A}_T(E)$ has positive Lebesgue measure when the Hausdorff dimension of E is greater than $3/2$. We note that the threshold for trees is independent of the number of edges, whereas for the rigid case above, the threshold increases with k. Furthermore, through a projection argument, we prove that the similar tree problem reduces to the case of a single edge, giving the following result: the collection of similar area-tree configurations by scale factor r has positive Lebesgue measure when E has Hausdorff dimension greater than $3/2$.

Induced-Saturation with Star Graphs

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Jared E Glassband (jeg328@cornell.edu)
Moravian University [Mentor:Kathleen Ryan]

Abstract of Report Talk: Using the definition of Tennenhouse, a graph G is *induced H-saturated* if there exists no induced subgraph H in G, but for every edge $e \in \overline{G}$, $G+e$ has an induced subgraph H. Tennenhouse showed the existence of induced $K_{1,3}$-saturated graphs for $n \geq 12$. Inspired by his results for this star graph $K_{1,3}$, we show there exist induced $K_{1,3}$-saturated graphs on n vertices if and only if $n \geq 8$. Additionally, we construct arbitrarily large induced $K_{1,m}$-saturated graphs for $4 \leq m \leq 7$ by adopting techniques from Behrens, Erbes, Santana, Yager, and Yeager. Finally, for the double star $D_{2,2}$, we show via a constructive proof that there exists an induced $D_{2,2}$-saturated graph on n vertices if and only if $n \geq 12$. [NM29144512]

[Joint with Kevin Hua, Evan Sabini, Brian Kronenthal]

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Polychromatic Ramsey Properties of Ideals

Jonah A Nan (jonahnan@umich.edu)
University of Michigan, Ann Arbor [Mentor:Zach Norwood]

Abstract of Report Talk: Ramsey's theorem states that if pairs of natural numbers are colored using finitely many colors, then there is an infinite monochromatic subset. We consider for which ideals I on the natural numbers and for which n the "infinite monochromatic" clause can be modified to "I-positive n-chromatic". The case $n = 1$ was studied by Hrusak et al. (2017). We generalize this to consider arbitrary values of n. We introduce a family of ideals $ED_{m,n}$ which exhibit nice polychromatic Ramsey properties. We prove exactly which Ramsey properties for colorings of pairs imply which others. We also consider generalizing from colorings of pairs to colorings of k-element sets. [NJ06153123]

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Take-away Impartial Combinatorial Game on Different Geometric and Discrete Structures

Molena Nguyen (molena.thy.nguyen@gmail.com)
Berea College [Mentor:Kristen Barnard]

Abstract of Report Talk: Following from the winning strategy for a Take-Away Impartial Combinatorial Game on only Oddly Uniform or only Evenly Uniform Hypergraphs in the Ph.D. Dissertation of Dr. Kristen Barnard (an Assistant Professor of Mathematics at Berea College), Molena Nguyen found the new winning strategy for a Take-Away Game on neither Oddly nor Evenly Uniform Hypergraphs during her Undergraduate Independent Research opportunity. However, these neither Oddly nor Evenly Uniform Hypergraphs must meet the given special requirements. In a Take-Away Game on hypergraphs, two players take turns to remove the vertices and the hyperedges of a hypergraph. In each turn, a player must remove either only one vertex or only one hyperedge. When a player choses to remove one vertex, all of the hyperedges that contain the chosen vertex are also removed. When a player choses to remove one hyperedge, only that one chosen hyperedge is removed. Whoever removes the last vertex wins the game. All of the new theorems in this research paper are in agreement with the previous theorems in the Ph.D. Dissertation of Dr. Kristen Barnard. **[NM17020340]**

[Joint with N/A] Received: August 16, 2021

Curves in Non-Orientable Surfaces

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Georgia Tech [Mentor:Dan Margalit]

Abstract of Report Talk: A well studied problem in topology is to find collections of nontrivial curves in orientable surfaces with certain intersection properties. For example, Malestein, Rivin, and Theran showed that there exists at least $g^2 + \frac{5}{2}g$ curves in a genus g surface that pairwise intersect 0 or 1 times. We can look for similar collections of curves in non-orientable surfaces. A surface is *non-orientable* if and only if it contains a Möbius band. The Klein bottle and the Real projective plane are basic examples of non-oreintable surfaces. Because non-orientable surfaces contain torsion, curve behaviors are counterintuitive. We generalize Malestein, Rivin, and Theran's construction to non-orientable surfaces and exhibit a lower bound for the maximum number of curves that pairwise intersect 0 or 1 times in a generic non-orientable surface. **[NS28110109]**

[Joint with Nancy Scherich] Received: August 19, 2021

Building Young Tableaux from Extended Sets

Eric S Nofziger (enofzige@butler.edu)
Butler University [Mentor:Amber Russell]

Abstract of Report Talk: Young tableaux are combinatorial objects related to the partitions of an integer and have various applications in representation theory. They are particularly useful in the study of the fibers arising from the Springer resolution. The study of this map was instrumental in the classification of irreducible representations of finite groups of Lie type, and it is still a vibrant source of ongoing research. In recent work of Graham–Precup–Russell, an association has been made between a given row-strict tableau and three disjoint subsets I, J, and K, also called extended sets. These subsets are then used in the study of extended Springer fibers. In this project, we begin to classify which extended sets correlate to a valid row-strict or standard tableau. We are able to identify several global properties of these valid sets, and we further find an algorithm that produces a valid tableau given only the extended sets in special cases. **[NE02181732]**

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A New Graph Invariant Between Scramble Number and Gonality

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Williams College [Mentor:Ralph Morrison]

Abstract of Report Talk: Divisorial gonality is a combinatorial graph invariant with important implications in divisor theory on algebraic curves. Two other graph parameters, bramble number and scramble number, are known to lower bound gonality, with scramble number being the strictly better bound (Harp et. al, 2020). However, bramble number is better understood because of its dual characterization in terms of treewidth, a fundamental graph invariant defined as the minimum width of a tree decomposition. An analogous notion of treewidth to complement scramble number is thus a promising path toward greater insight into this powerful bound.

Building off prior work on tree-cut decompositions, we introduce *scramble-cut width*, as such an analogous partner to scramble number. We provide a graph-theoretic proof that scramble-cut width upper bounds scramble number, giving the only known method for sharply upper bounding scramble number when it is strictly less than gonality. Using the theory of graph divisors, we present a strategy for constructing an explicit tree-cut decomposition which will hopefully lead to a proof that scramble-cut width lower bounds gonality. We then provide examples of graphs with scramble-cut width strictly greater than scramble number, exemplifying that scramble-cut width could indeed provide a better lower bound on gonality.
[OL06101701]

[Joint with Shefali Ramakrishna, Jason Meintjes, Lizzie Ferguson, Eyobel Gebre, Cassandra Marcussen, Ben Weber]

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A Twisted Thistlethwaite Theorem

Joseph D Paugh (paugh.30@osu.edu)
The Ohio State University [Mentor:Sergei Chmutov]

Abstract of Report Talk: Elegantly connecting knot theory and graph theory, Thistlethwaite's celebrated 1987 theorem constructs a planar graph from an alternating link diagram, producing a correspondence between the Jones polynomial of knot theory and the Tutte polynomial of graph theory. The Jones polynomial was later generalized to an invariant of virtual links - the arrow polynomial - in a 2009 paper of Dye and Kauffman, useful for its applications in bounding the virtual crossing number. Then in 2012, an analogue of Thistlethwaite's theorem for the arrow polynomial was produced by Chmutov et al, obtained by constructing a ribbon graph (a representation of a graph embedding) from a virtual link diagram. In particular, the result takes a generalized Tutte polynomial for ribbon graphs - the arrow dichromatic polynomial - and specializes it to the arrow polynomial of a virtual link diagram. In the same spirit, our research group developed a ribbon graph construction for twisted links: abstract link diagrams embedded in potentially non-orientable surfaces. The result is a Thistlethwaite-type theorem for twisted links, directly generalizing the result of Chmutov et al by relating the arrow dichromatic polynomial to the arrow polynomial of a twisted link diagram. **[PJ04205147]**

[Joint with Justin Wu, Gavin Zhang] Received: August 17, 2021

Topological Properties of Complex Almost Abelian Lie Groups

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University of California, Santa Barbara [Mentor:Zhirayr Avetisyan]

Abstract of Report Talk: If a Lie group is non-Abelian but has a codimension 1 Abelian subgroup, we say that the Lie group is almost Abelian. We show that all discrete subgroups of complex simply connected almost Abelian groups are finitely generated. The topology of connected almost Abelian Lie groups is studied by expressing each connected almost Abelian Lie group as a quotient of its universal covering group by a discrete normal subgroup. We then prove that no complex connected almost Abelian group is compact, and give conditions for the compactness of connected subgroups of such groups. Towards studying the homotopy type of complex connected almost Abelian groups, we investigate the maximal compact subgroups of such groups. **[PA04201744]**

[Joint with Oderico-Benjamin Buran] Received: August 17, 2021

Walking to Infinity Along Some Number Theory Sequences

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Williams College [Mentor:Steven Miller]

Abstract of Report Talk: An interesting open conjecture asks whether it is possible to walk to infinity along primes, where each term in the sequence has one digit more than the previous. We present different greedy models for prime walks to predict the long-time behavior of the trajectories of orbits, one of which has similar behavior to the actual backtracking one. Furthermore, we study the same conjecture for square-free numbers, which is motivated by the fact that they have a strictly positive density, as opposed to primes. We introduce stochastic models and analyze the walks' expected length and frequency of digits added. Lastly, we prove that it is impossible to walk to infinity in other important number-theoretical sequences, namely, perfect squares and Fibonacci numbers, or on primes in different bases. **[PF05085325]**

[Joint with Nawapan Wattanawanichkul, Joshua Siktar]

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Complexity of the Zeckendorf Graph Game

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Williams College [Mentor:Steven Miller]

Abstract of Report Talk: Every integer $n > 0$ can be written uniquely as a sum of non-adjacent Fibonacci numbers $\{1, 2, 3, 5, \dots\}$, called its Zeckendorf decomposition. For example, $80 = 1 + 3 + 21 + 55 = F_1 + F_3 + F_7 + F_9$. Earlier work analyzed a two-player combinatorial game played on an infinite directed path graph; starting with n chips at the 1st vertex v_1, players alternate by playing two types of moves arising from the Fibonacci recurrence. The game must terminate in Zeckendorf decomposition of n, and the last player to move wins. For example, the game beginning with 80 chips terminates as follows:

$$(1) \to (0) \to (1) \to (0) \to (0) \to (0) \to (1) \to (0) \to (1) \to \dots$$

The second player to move has a winning strategy, but the proof is a strategy stealing argument and therefore non-constructive. Despite efforts by dozens of students and mentors over the last 3 years, no past investigation has identified a winning strategy.

We give new evidence that solving the Zeckendorf Game is algorithmically hard. We generalize the Zeckendorf Game to a slightly broader game, which we then show is PSPACE-hard. One may think of the original game as a two player pebble game on the infinite path, and we expand the game to being played on any directed acyclic graph. We then show that on planar leveled digraphs, the game terminates within polynomial time (whereas it does not on general digraphs). Our proof leverages a natural projection of the game on a leveled graph to a corresponding path game, and we then rely on previous results that the Zeckendorf Game terminates in polynomial time.

From there, we may conclude that Planar-Leveled-ZGG (PLZGG) is in PSPACE, meaning it can be solved using an amount of memory polynomial in the number of initial chips and the size of the graph. A problem is PSPACE-complete if any problem in PSPACE can be reduced to an instance of the problem in question. By reducing another PSPACE-complete problem called the formula game to PLZGG, we prove that PLZGG is also PSPACE-complete. As such, we amass evidence that the Zeckendorf Game may be intractable to solve. We also generalize our results to a much broader class of novel PSPACE-complete games which arise from linear recurrences. **[PE06144931]**

[Joint with Irfan Durmic, Henry Fleischmann, Faye Jackson, Luke Reifenberg] Received: August 18, 2021

Kantorovich duality formulation from discrete to general and the Wasserstein quotient space

Nhung Pham (pham0306@umn.edu)
UCLA [Mentor:Wilfrid Gangbo]

Abstract of Report Talk: Optimal Mass Transportation was first formulated by Gaspard Monge in 1781 and has gained a lot of attention and research effort for the last three decades. Optimal Transport plays an important role in partial differential equations, fluid mechanics, probability theory and functional analysis. Kantorovich later proposed a relaxed version of Monge problem and identified a dual problem. The goal of our project is to produce a duality proof based on arguments accessible to undergraduate level students. Our starting point is a 2018 paper by Haim Brezis who gave an elementary proof to the coincidence in the discrete settings of the three quantities: the original minimal cost introduced by Monge, the relaxed version by Kantorovich and the duality formulation also due to Kantorovich. This research deduced that the whole transport duality theory can be derived from the elementary arguments of Brezis. The project rests on approximating any probability measure by a sequence of averages of Dirac masses, so that the discrete result implies the general coincidence. In addition, the research studied the quotient of the Wasserstein space by the group of rigid transformations. **[PN30122023]**

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Automorphisms of the fine curve graph

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Georgia Institute of Technology [Mentor:Margalit Dan]

Abstract of Report Talk: The curve graph of a surface is a graph whose vertices are isotopy classes of simple, closed, essential curves on the surface, and there exists an edge between two vertices if the two isotopy classes are disjoint. Ivanov [1] showed that the automorphism group of the curve graph is isomorphic to the extended mapping class group, the group of isotopy classes of all homeomorphisms of a surface. We extend Ivanov's work by considering the fine curve graph of a surface (defined by Bowden, Hensel, and Webb [2]), where vertices of the graph are all curves on the surface, and there exists an edge between two vertices if the curves are disjoint. Similar to Ivanov's work, we show that the automorphism group of the fine curve graph is isomorphic to the extended mapping class group. This is joint work with Dr. Dan Margalit and Dr. Yvon Verberne. This project stems from work completed at the Georgia Tech REU. [1] J. Bowden, S. Hensel, and R. Webb. Quasi-morphisms on surface diffeomorphism groups. arXiv:1909.07164, 2019. To appear in J. Amer. Math. Soc. [2] Nikolai V. Ivanov. Automorphism of complexes of curves and of Teichmüller spaces. Internat. Math. Res. Notices, (14):651–666, 1997. **[PA22160037]**

[Joint with Adele Long, Yvon Verberne]
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On Kostant's weight q-multiplicity formula for $\mathfrak{sp}_6\mathbb{C}$

Daniel C Qin (dcqin17@gmail.com)
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Williams College [Mentor:Pamela Harris]

Abstract of Report Talk: Kostant's weight q-multiplicity formula is an alternating sum over a finite group known as the Weyl group and whose terms involve the q-analog of Kostant's partition function, which is a polynomial-valued function defined by $\wp_q(\xi) = \sum_{i=0}^{k} c_i q^i$, where c_i is the number of ways the weight ξ of $\mathfrak{g}$ can be written as a sum of exactly i positive roots of $\mathfrak{g}$. The evaluation of the q-multiplicity formula at $q = 1$ recovers the multiplicity of a weight in a irreducible highest weight representation of $\mathfrak{g}$. In this presentation, we specialize to the Lie algebra $\mathfrak{sp}_6(\mathbb{C})$ and we provide a closed formula for the q-analog of Kostant's partition function, which extends recent results of Shahi, Refaghat, and Marefat. We also describe the supporting sets of the multiplicity formula (known as the Weyl alternation sets of $\mathfrak{sp}_6(\mathbb{C})$), and use these results to provide a closed formula for the q-multiplicity for any pair of dominant integral weights of $\mathfrak{sp}_6(\mathbb{C})$. Throughout this work, we provide code to facilitate these computations. **[QD05140505]**

[Joint with Peter Hollander]

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Speeding Up Gonality Computation

Shefali Ramakrishna (taxicabnumber1729@gmail.com)
Williams College [Mentor:Ralph Morrison]

Abstract of Report Talk: Chip-firing games are a combinatorial tool used in divisor theory on algebraic curves. Gonality is a key invariant for both graphs and curves, and measures the minimum degree of a positive-rank divisor. One can determine the gonality of a graph through brute-force checks of every possible divisor, a very inefficient process. In fact, computing gonality is known to be NP-hard (Gijswijt et al., 2020), so we need innovative tools to solve this problem.

We design and implement algorithms that provide speed-ups in gonality computation. This allows us to compute the gonality of graphs of arbitrary size subject only to the algorithm's run time, not to device memory. We use a lexicographic enumeration of divisors in our search, effectively cutting down the memory usage for storing divisors from n^d to $n \cdot \log d$ for degree d on a graph of n vertices. We also use efficiently checkable properties of partial orientations on graphs (Backman, 2014) to rule out divisors. In addition to computing gonalities for many specific graphs, we have used our implementation to provide the base case for a proof by induction characterizing the gonality of all Harary graphs $H(4, n)$. **[RS05115404]**

[Joint with Lisa Cenek, Lizzie Ferguson, Eyobel Gebre, Cassandra Marcussen, Jason Meintjes, and Liz Ostermeyer]

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Anti-Ramsey Multiplicities for Various Families of Graphs

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the Polymath Jr. Program [Mentor:Zhanar Berikkyzy]

Abstract of Report Talk: Suppose we have a fixed graph H and a large complete graph K_n. One can ask: how can we color the edges of K_n with red and blue in order to minimize the number of monochromatic copies of H in K_n, as $n \to \infty$? This minimum is called the Ramsey multiplicity of H. If the minimum is achieved by a uniformly random 2-edge-coloring, H is called common, and otherwise it is called uncommon. Common graphs have been studied extensively (see for example, Burr and Rosta, *On the Ramsey Multiplicity of graphs - problems and recent results* (1980)).

During the Polymath Jr. REU program this summer, we investigated the *rainbow* version of this problem. A graph is rainbow if each edge has a distinct color. We ask: given a graph H, how can we color the edges of K_n with r colors in order to *maximize* the number of *rainbow* copies of H in K_n, as $n \to \infty$? This maximum is known as the anti-Ramsey multiplicity of H. If the maximum is achieved by a uniformly random r-edge-coloring, we call H r-rainbow-common, and otherwise it is called r-rainbow-uncommon. Recently De Silva et al. in *Anti-Ramsey multiplicities* (2019) showed that stars and disjoint unions of stars are r-rainbow-common. They also established some properties of rainbow-uncommon-graphs. We will discuss our work in determining the anti-Ramsey multiplicities for cycles, paths, and other classes of graphs, as well as general statements regarding monotonicity of rainbow-commonality and subgraph relationships. **[RI08093447]**

[Joint with Gabriel Elvin, Polymath Jr. student researchers]

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The Delta-Unlinking Number of Algebraically Split Links

Moises Reyes Rivas (reyesrivas@andrews.edu)
Andrews University [Mentor:Anthony Bosman]

Abstract of Report Talk: It is known that algebraically split links (links with vanishing pairwise linking number) can be transformed into the trivial link by a series of local moves on the link diagram called delta-moves; we introduce the delta-unlinking number, defined to be the minimum number of such moves needed. This generalizes the well-studied delta-unknotting number for knots to algebraically split links of arbitrarily many components. We prove that the delta-unlinking number is bounded below by half the unlinking number, the 4-genus, and the slice genus and (for proper links) has the same parity as the Arf invariant. We note relationships with other classical link invariants, such as the Milnor mu bar invariants, and using these determine the delta-unlinking number and tabulate the minimal delta-pathway for prime algebraically split links with up to 9 crossings, as well as determine the 4-genus for most of these links. We generalize several of the relationships to the delta-Gordian distance between proper links, i.e. the minimal number of delta moves needed to move one proper link into another. **[RM21144752]**

[Joint with Jeannelle Green, Gabriel Palacios, Noe Reyes]

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Graph Coloring as a measure Network Reliability

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Mason H Nakamura (mason.nakamura1@marist.edu)
Moravian University [Mentor:Nathan Shank]

Abstract of Report Talk: Given a network, which we represent using a simple graph, one can assign a measure of reliability as the minimum number of alterations required to force the network to achieve some desired failure state. This makes networks that require n number of alterations to fail less reliably than a network that requires $n+1$ alterations to fail. Network reliability has previously been studied extensively on graph properties such as the diameter and the order under removal of edges and vertices. However, we analyze reducing the chromatic number and chromatic index under vertex and edge removals as a way to measure network reliability. Specifically, we say a network has failed when a proper coloring can be done with at most k colors. This gives 4 ways to see how reliable a network is for fixed k (2 degrees of freedom from choosing edge or vertex coloring, and 2 degrees of freedom from choosing vertex or edge removal). We then attempt to find the least reliable graphs in $G(n, m)$, the set of graphs with n vertices and m edges, with respect to each of these 4 network reliability parameters. **[RR06125156]**

[Joint with Mia Destefano, Grace M Mulry] Received: August 17, 2021

Methods for Solving the Family of Diophantine Equations $ax^2 - p^{2m} = y^n$

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Williams College [Mentor:Eva Goedhart]

Abstract of Report Talk: Expanding upon the work of Bugeaud (1997), we examine the finiteness of the number of integer solutions to the family of Diophantine equations

$$ax^2 - p^{2m} = \pm y^n,$$

where p is an odd prime. In this presentation, we introduce the methods used, including p-adic distances and linear forms in logarithms. Applying these techniques, we are able to show there are finitely many integer solutions to this equation, and demonstrate how our approach breaks down in a special case when the exponent on the prime is odd. **[RE03144629]**

[Joint with Brian Crane] Received: August 17, 2021

On generation, structure, and symmetries of minimal prime graphs.

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Texas State University [Mentor:Thomas Keller]

Abstract of Report Talk: The prime graph of a finite group G has vertices consisting of the prime divisors of the order of G and an edge from primes p to q if and only if G contains an element of order pq. We focus on a subclass of the prime graphs of finite solvable groups, called minimal prime graphs. A minimal prime graph's complement is triangle-free and 3-colorable, along with the property that removing an edge violates 3-colorability or creates a triangle in the complement. We introduce clique generation, a novel method to construct minimal prime graphs. Then, we prove that the tensor products of graphs can be used to construct prime graphs of solvable groups but not minimal prime graphs. In addition, we prove a connection between the automorphism groups of a minimal prime graph and some of its generated graphs. Finally, we study how the structure of a minimal prime graph restricts possible Frobenius actions in a group that realizes this prime graph. **[RM06165840]**

[Joint with Ziyu Huang, Wen Plotnick] Received: August 18, 2021

Bijections on convex sets and closed convex projective surfaces that preserve complete geodesics

Drimik Roy Chowdhury (drimikr@umich.edu)
University of Michigan, Ann Arbor [Mentor:Ralf Spatzier and Giuseppe Martone]

Abstract of Report Talk: This question has been investigated in the contexts of n-dimensional Euclidean space, hyperbolic space and on the flat torus. We will discuss some history in these settings. Our work has been focused on asking the same question but to greater generalities such as bounded, open convex sets and closed convex projective surfaces, which are both endowed with the Hilbert metric. In short, a few current goals in mind are to hopefully understand the local behavior of such maps (e.g. are they continuous? do they preserve laminations?) and to what extent they determine the topology (e.g. does our map preserve closed geodesics? simple geodesics?). We will provide some preliminary results to these questions. **[RD06154525]**

[Joint with another mentor: Giuseppe Martone] Received: August 18, 2021

A Sharp Bound on the Maximum Number of Independent Transversals

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()
Baruch College, CUNY [Mentor:Fan Wei]

Abstract of Report Talk: Let G be a graph and $V_1 \cup V_2 \cup \ldots \cup V_k$ partition $V(G)$. Let $d_{i,j}$ denote the edge density of the pair (V_i, V_j). An independent transversal is an independent set of G with exactly one vertex in each V_i. In this paper, we prove an asymptotically sharp upper bound for the maximum number of independent transversals given the $d_{i,j}$'s. Bounding the number of independent transversals is a common theme in extremal combinatorics. It often appears in inducibility type results for many combinatorial structures such as permutations and graphs. This paper combines ideas and techniques from various subfields of mathematics: optimization theory plays a key role in the proof of our main result and we discuss some stability results which have an analytic flavor. **[RJ06161108]**

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Main Modes of Microfilament Particles Deformation in Rough Channels

Alexandra Sabrio (aasabrio@crimson.ua.edu)
University of Alabama [Mentor:Mojdeh Rasoulzadeh]

Abstract of Report Talk: Microfiber filaments are the most common type of microplastic particles polluting the environment and are recognized as a growing threat to groundwater resources and human health. A better understanding of how microfibers move, deform, and settle in the preferential pathways of fluids in subsurface systems would be beneficial, aid in the improvement of current soil remediation procedures, and advance waste disposal management techniques.

We investigate the transport and deformation of a single microfilament in a channel with rough boundaries. Previous research has focused on colloidal particles, but this fails to account for the path of microfilaments and their main modes of deformation. An immersed boundary method is used to model fluid flow within a rough channel and the transport of microfilament through the channel. The microfilament is modeled as a set of beads connected by beam elements. A system of ODEs for dynamics of motion and deformation of microfilament is coupled to full set of Navier Stokes equations for fluid flow. The filament's path and deformation are recorded; furthermore, channel roughness is altered while maintaining fluid flow at various low and high Reynolds numbers. Roughness profile amplitude and wave length are key parameters that affect the deformation modes of the microfilament particle. In addition to roughness, the structure of path lines can greatly influence the deformation of microfilaments. At low Reynolds numbers, with more significant viscous effects, the microfilament is transported along paths that are parallel to rough boundaries and undergoes moderate deformations. Increasing the Reynolds number results in more important inertial terms, formation of vorticities and flow instabilities, and major deformations and local circulations of the microfilament. **[SA29110428]**

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An Algorithmic Comparison of Pneumonia Locating and Severity Analysis

Axel Sanchez Moreno (a_s1350@txstate.edu)
Texas State University [Mentor:Ivan Ojeda-Ruiz]

Abstract of Report Talk: Radiologists determine whether a patient needs to be placed into an Intensive Care Unit (ICU) by manually tracing pneumonia from the Computerized Tomography (CT) scans of COVID-19 patients. However, tracing through each frame is time-consuming and will lead to human error, so we plan to compare the Normalized Cut (NCut) and k-means algorithms to determine which is more effective in determining the location and severity of a patient's pneumonia. When analyzing the quality of a segmentation, we computed the values of 2 validation metrics: accuracy and dice. We hope this will support radiologists in embracing image segmentation and a tool for locating a patient's pneumonia and measuring its severity in a short time frame. **[SA05164619]**

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[Mentor:]

Abstract of Report Talk: **[]**

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A Novel Generalization of Diophantine m-tuples in Finite Fields

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Queen's University [Mentor:Seoyoung Kim]

Abstract of Report Talk: Let S be a set of m positive integers $\{a_1, a_2, \ldots, a_m\}$ with the property that $a_i a_j + 1$ is a perfect square for $1 \leq i < j \leq m$. We call S a Diophantine m-tuple. A natural question arises when one studies these sets: how many Diophantine m-tuples exist for any arbitrary value of m? Euler found an infinite collection of Diophantine quadruples, and He, Togbé and Ziegler showed that a Diophantine quintuple does not exist. However, this question has not received as much attention over other commutative rings with unity. For instance, Dujella and Kazalicki only recently found the number of Diophantine m-tuples in the finite field $\mathbb{F}_p$ for $m = 2, 3, 4$.

We introduce a new generalization of the Diophantine m-tuples. A k-Diophantine m-tuple is a set of non-zero elements of a given commutative ring with unity such that every k-wise product of distinct elements is one less than a perfect square. We study these in finite field $\mathbb{F}_p$ where p is prime. In an approach analogous to that of Dujella and Kazalicki, we use Legendre sums to derive a formula for the number of 3-Diophantine triples in $\mathbb{F}_p$, viz.

$$N^{(3)}(p) = \begin{cases} \frac{a+1}{3} + \frac{\binom{p-1}{3}}{2}, & \text{for } p \equiv 1 \pmod 3 \\ \frac{\binom{p-1}{3}}{2}, & \text{for } p \equiv 2 \pmod 3 \end{cases}$$

where a is an integer such that $a \equiv 2 \bmod 3$ and $p = a^2 + 3b^2$ for some integer $b > 0$. Moreover, the methodology for counting the number of 3-Diophantine m-tuples should generalize to k-Diophantine m-tuples. Finally, we prove that there exists at least one k-Diophantine m-tuple in $\mathbb{F}_p$ for sufficiently large p, namely

$$p > 4\left(4^{\binom{m}{k-1}}\left(\frac{\binom{m}{k-1}}{2} + m + 1\right)^2\right).$$

[SL05114309]

[Joint with Kyle Onghai, Arjun Nigam]

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The Structural Incidence Problem for Lattices

Junxuan Shen (jshen@caltech.edu)
CUNY Baruch College [Mentor:Adam Sheffer]

Abstract of Report Talk:

We prove new structural results for point-line incidences. An incidence is a pair of one point and one line, where the point is on the line. The Szemerédi-Trotter theorem states that n points and n lines form $O(n^{4/3})$ incidences. This bound has been used to obtain many results in combinatorics, number theory, harmonic analysis, and more. While the Szemerédi-Trotter bound has been known for several decades, the structural problem remains wide-open. This problem asks to characterize the point-line configurations with $\Theta(n^{4/3})$ incidences.

We completely characterize the case where the point set is a lattice.

Theorem. *Let $1/3 < \alpha < 2/3$. Let $\mathcal{P}$ be a lattice of size $n^{\alpha} \times n^{1-\alpha}$ and $\mathcal{L}$ be a set of n lines, such that $\mathcal{P}$ and $\mathcal{L}$ form $\Theta(n^{4/3})$ incidences. Then $\mathcal{L}$ contains $\Omega(n^{1/3}/\log n)$ families of $\Theta(n^{2/3})$ parallel lines. The y-intercepts of each family of parallel lines form an arithmetic progression.*

When $\alpha < 1/3$ or $\alpha > 2/3$, it is impossible to have $\Theta(n^{4/3})$ incidences. We also obtain similar results where $\mathcal{P} = A \times B$ where only one of A, B is an arithmetic progression. That is, in the case where only one axis of the Cartesian product $\mathcal{P}$ behaves like a lattice. **[SJ04210815]**

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Markov chains analyses of abstract strategy games and expansion of game boards to Platonic solids

Xiaoning Shi (xshi229@wisc.edu)
Hofstra University [Mentor:Johanna Franklin]

Abstract of Report Talk: An abstract strategy game is a game with no element of chance (the "strategy" part) and no underlying story (the "abstract" part), such as Tapatan and Picaria. On a standard Tapatan board, each edge of the square is labeled with a midpoint, the square has a center, which is connected to all midpoints and vertices. Two players each have three stones with the goal of making a row. The standard Picaria board builds on the above and connects all adjacent midpoints in pairs. However, in practical play, there are also random elements in abstract strategy games. Markov chains model scenarios where the probability of occurrence of future state depends only on the present state, by which we analyze games of skill in a probabilistic way. Further, these abstract strategy game boards can be expanded to a higher dimensional space. In three-dimensional space, a Platonic solid is a convex polyhedron in which all faces are identical in shape and size and the number of faces connected to each vertex is the same. We expand the standard Tapatan and Picaria boards to Platonic solids by building 3D models. After discussing and calculating the winning configurations for each board in separate situations, a total winning formula that applies to all boards is concluded.
[SX06062300]

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Spectral Analysis of the Kohn Laplacian on Lens Spaces

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University of Michigan-Dearborn [Mentor:Yunus Zeytuncu]

Abstract of Report Talk: A key problem in spectral geometry is the relationship between the spectrum of the Laplacian and the geometry of the underlying manifold. For example, the celebrated Weyl's law relates the asymptotics of the spectrum to the volume of the manifold. Analogous to the Laplace-Beltrami operator on Riemannian manifolds, we study the Kohn Laplacian for functions on CR manifolds. Lens spaces are manifolds obtained as quotients of the sphere $S^{2n-1} \subset \mathbb{C}^n$ by cyclic unitary actions with the natural CR structure inherited from the spheres. Folland computes the eigenvalues of the Kohn Laplacian on S^{2n-1} and these eigenvalues descend on the quotient with different multiplicities. We investigate these multiplicities on lens space by analyzing a system of Diophantine equations. Using this, we prove an analog of Weyl's law for the Kohn Laplacian on lens spaces.

While Ikeda and Yamamoto classify isospectral lens spaces as Riemannian manifolds, an analog for the Kohn Laplacian on lens spaces does not exist in the literature. We conjecture that two 3-dimensional lens spaces are isospectral if and only if they are CR isometric, and we present several partial results in this direction. In particular, we show that two 3-dimensional lens spaces with fundamental groups of equal prime order are isospectral if and only if they are CR isometric. Since CR isometry is a stronger condition than Riemannian isometry, this implies that the spectrum of the Kohn Laplacian contains more information than the spectrum of the Laplace-Beltrami operator. In particular, we present lens spaces that are isospectral with respect to the Laplace-Beltrami operator, but have different spectra with respect to the Kohn Laplacian, reflecting the fact that information on the CR structure is contained in the CR spectrum. **[SI05214603]**

[Joint with Zoe Plzak, Colin Fan, Elena Kim]

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Structural Szemeredi–Trotter for non-Lattices

Olivine Silier (osilier@caltech.edu)
MIT [Mentor:Lawrence Guth]

Abstract of Report Talk: The study of point-line incidences was initiated by Erdős. A point and a line form an *incidence* if the point is on the line. For $|P| = n$ and $|L| = m$, the Szemerédi–Trotter theorem states that the number of incidences between points from the set P and lines from the set L is $O(n^{2/3}m^{2/3})$. Though the Szemerédi–Trotter theorem has been known for four decades, hardly anything is known about the *structural problem*: characterizing configurations with $\Theta(n^{2/3}m^{2/3})$ incidences. So far only configurations with an underlying lattice structure have been known to have a maximal number of incidences. In this work we use a number theoretic approach to construct a family of configurations where the point set is not a lattice, but rather a Cartesian product of a generalized arithmetic progression, and the number of incidences is maximal. **[SO07184509]**

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Optimization of Spin-Error-Correction in Finite-Dimensional Hilbert Spaces

Aditya D Sivakumar (asivakum@caltech.edu)
California Institute of Technology [Mentor:Victor Albert]

Abstract of Report Talk: Information can be encoded in finite-dimensional subspaces of a quantum Hilbert space. A disadvantage of these information systems is that the information can be easily damaged by noise sources. The errors caused by this noise are represented as linear endomorphisms on the given Hilbert space. Fortunately, there often exist subspaces of the Hilbert space within which any stored information cannot be irrecoverably lost due to the action of these endomorphisms. It turns out that for the information to be fully recoverable, all that is required is for the restriction of any composition of two such endomorphisms to the subspace to be a diagonalizable linear transformation with all its eigenvalues equal to each other. In this paper, we focused on finite-dimensional Hilbert spaces affected by a specific type of noise: the action of SU(2) by the quantum spin representation. We conjectured that a type of subspace known as a binomial code would protect any stored information from this noise. We then developed a rigorous analytic proof to support this claim. **[SA06161400]**

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An Explicit and Analytical Solution to the Stokes Equations on the Half-Space

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Oregon State University [Mentor:Hoe Woon Kim]

Abstract of Report Talk: We present explicit and analytical solutions to the Stokes equation on the half-space with initial and boundary data for velocity. Our approach to a solution for velocity and pressure involves integral transformations, Green's functions, and the Helmholtz Decomposition Theorem. Our main results demonstrate that the velocity derived from this initial and boundary value problem can be expressed as the curl of the convolution of vorticity and the fundamental solution to the Laplace equation in our domain. **[SM06164633]**

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A Stochastic Central Limit Theorem and Applications to Integer Decompositions

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Williams College [Mentor:Steven Miller]

Abstract of Report Talk: Base-b decompositions have long been known and valued. They were first extended to non-integer bases in 1953 by George Bergman when he showed that every positive integer n can be expressed as a sum of integer powers of the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$ using at most one copy of each power. If we add the additional rule that no two consecutive powers can be used, each integer has a *unique* representation. This gives rise to a number system with an irrational base, where, for example, $4 = \varphi^2 + \varphi^0 + \varphi^{-2} = 101.01$.

In our paper, we consider the number of 1's in the base-φ representation of an integer n. We show that the distribution of the number of 1's needed to represent $n \in [L_m, L_{m+1})$, where L_m is the m^{th} Lucas number defined by the same recurrence as the Fibonacci numbers but with initial conditions $L_0 = 2, L_1 = 1$, approaches a Gaussian in probability and we compute the k^{th} mean and variance. We prove these results by deriving a, possibly new, central limit theorem for stochastic processes. Furthermore, we note that a condition similar to regenerativity is also satisfied, and could be studied independently of our results.

Earlier authors have studied decompositions into summands of the form $a_1H_1 + \cdots + a_nH_n$, where H_n is a sequence generated by a positive linear recurrence, that is, $H_n = c_1H_{n-1} + \cdots + c_LH_{n-L}$. In *From Fibonacci Numbers To Central Limit Type Theorems*, Miller and Wang showed that there exist unique *reduced* decompositions of the listed form. They also proved that the number of summands of reduced decompositions over $[H_n, H_{n+1})$ tends to a Gaussian with linear mean and variance. This required a monumental combinatorial effort involving numerous special cases of associated characteristic polynomials. Our central limit theorem allows us to recover their result with considerably less effort. In particular, we showcase a short proof of Lekkerkerker's Theorem that the average number of 1's in the Zeckendorf decomposition in the Fibonacci interval $[F_n, F_{n+1})$ is asymptotic to $\frac{n}{\varphi^2+1}$.

[SA05213328]

[Joint with Irfan Durmic and Alicia Smith Reina]

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Preferential and k-Zone Parking Functions

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MSRI-UP 2021 [Mentor:Pamela E. Harris]

Abstract of Report Talk: Parking functions are vectors that describe the parking preferences of n cars that enter a one-way street containing n parking spots numbered 1 through n. A list of each car's preferences is also compiled into vectors in which we denote as $(a_1, \ldots, a_n)$, such that a_i is the parking preference for car i. The classical parking rule allows cars to enter the street one at a time going to their preferred parking spot and parking, if that space is unoccupied. If it is occupied, they then proceed down the one-way street and park in the first available parking spot. If all cars can park, we say the vector $(a_1, \ldots, a_n)$ is a parking function.

In our research, we introduce new variants of parking function rules with backward movement called k-Zone, preferential, and inverse preferential functions. We study the relationship between k-Zone parking functions and k-Naples parking functions and count the number of parking functions under these new parking rules which allow cars that find their preferred spot occupied to back up a certain parameter. One of our main results establishes that the set of non-increasing preference vectors are k-Naples if and only if they are k-Zone. For one of our findings we provide a table of values enumerating these new combinatorial objects in which we discover a unique relationship to the order of the alternating group A_{n+1}, numbers of Hamiltonian cycles on the complete graph, K_n, and the number of necklaces with n distinct beads for $n!$ bead permutations. **[SC06162412]**

[Joint with Pamela Vargas]

Received: August 20, 2021

Spinors and Graph Theory

Ethan Spingarn (espingarn23@amherst.edu)
()
Yale University [Mentor:Ivan Contreras Palacios]

Abstract of Report Talk: The evolution of quantum mechanical systems can be described by the Schrödinger equation, and the Dirac equation for spinors (particles with spin). In this talk, we study the behavior of spinors on a graph, as predicted by the Dirac equations, using analytic and algebraic methods. Analytically, we investigate the evolution of quantum states using graph-theoretic versions of the Schrödinger and Dirac equations, and we study discrete models of the Laplacian and Dirac operators. Algebraically, we study spinors as linear representations of the spin group using graph Clifford algebras. We also give gluing results on the centers of graph Clifford algebras. **[SE30114420]**

[Joint with Beata Casiday, Sabrina Mi]

Received: August 17, 2021

Modeling Wolbachia infected Anopheline mosquitoes for controlling Malaria in Haiti

Benjamin P Stager (bstager@tulane.edu)
Tulane University [Mentor:James Hyman]

Abstract of Report Talk: Malaria elimination is possible in Haiti. Preliminary studies of Anopheles (An.) mosquitoes infected with wAlbB Wolbachia bacteria show that infected mosquitoes are less capable of spreading Malaria. The infection induces a cytoplasmic incompatibility that disrupts the infection cycle through population suppression, inhibiting within-vector replication of the Plasmodium falciparum parasite, and reducing vector competence. Wolbachia is widespread among arthropods, and there are ongoing trials for sustaining wild populations of Wolbachia-infected Aedes aegypti to prevent the spread of Dengue fever. We create and analyze a model to evaluate different approaches for maintaining wAlbB infection within An. albimanus mosquitoes and apply it to assess its potential as a Malaria control strategy in Haiti. Our nine non-linear differential equation model divides the mosquito population by sex, infection status, pregnancy status and it includes an aquatic stage. The model simulates the combination of traditional Malaria vector control strategies of insecticide-treated bed net (ITN) distribution and indoor residual spraying (IRS) with different release scenarios of wAlbB-infected mosquitoes, and it evaluates the impact that seasonality has on achieving stable high-infection (90%) among wild An. albimanus mosquitoes. Preliminary findings indicate that use of IRS at baseline and release of infected pregnant female mosquitoes during the rainy season result in the fastest establishment of endemic wAlbB transmission. Consequently, acquiring endemic wAlbB infection can reduce Malaria transmission in An. mosquitoes and thereby be a potentially supplementary vector control strategy in elimination settings. **[SB03202736]**

[Joint with Alyssa J. Young, Daniela A. Florez, Kerlly J. Bernabé, Zhuolin Qu] Received: August 18, 2021

Quantitative Transversal Theorems in the Plane

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()
Baruch College, the City University of New York [Mentor:Pablo Soberon]

Abstract of Report Talk: Hadwiger's theorem is a variant of Helly-type theorems involving common transversals to families of convex sets instead of common intersections. In this presentation, we show a quantitative version of Hadwiger's theorem on the plane: given an ordered family of pairwise disjoint and compact convex sets in $\mathbb{R}^2$ and any real-valued monotone function on convex subsets of $\mathbb{R}^2$, if every three sets have a common transversal, respecting the order, such that the intersection of the sets with each half-plane defined by the transversal are valued at least (or at most) some constant α, then the entire family has a common transversal with the same property. Unlike previous generalizations of Hadwiger's theorem, we prove that disjointness is necessary for the quantitative case. We also prove colorful versions of our results. **[TY05221224]**

[Joint with Ilani Axelrod-Freed] Received: August 17, 2021

High-wavenumber steady solutions of two-dimensional Rayleigh–Bénard convection

Alexander R Takla (atakla@umich.edu)
University of Michigan [Mentor:Baole Wen]

Abstract of Report Talk: Rayleigh–Bénard convection (RBC)—a classical problem in fluid dynamics—plays a significant role in a large range of phenomena in engineering, geophysics, and astrophysics, and has been studied extensively to gain insights into the development of turbulence. Recent investigations show that steady solutions share many features with the turbulent RBC from direct numerical simulations and form the state space skeleton of the turbulent dynamics. This study focuses on the high-wavenumber, steady convection roll solutions that bifurcate supercritically from the motionless conductive state for two-dimensional RBC between stress-free boundaries. In order to elucidate the asymptotic behavior of the heat transport enhancement factor, i.e., the Nusselt number Nu, we compute steady convective rolls with aspect ratios $0.06 \leq \Gamma \leq \pi/5$ over six orders of magnitude in the Rayleigh number $10^8 \leq Ra \leq 10^{14}$ for fixed Prandtl number $Pr = 1$. Previous work indicates the dependence of Nu on Γ has a maximum at $\Gamma \approx 1.9$ at large Ra for low and moderate wavenumbers $k = 2\pi/\Gamma \leq 10$. Preliminary results of this study, however, show that there exists a second *local* maximum in the high-wavenumber regime where the aspect ratio Γ^*_{loc} that locally maximizes $Nu(\Gamma)$ scales as $Ra^{-1/4}$ and the corresponding Nu and Re scale as $Ra^{0.29}$ and $Ra^{2/5}$ respectively as $Ra \to \infty$. Nevertheless, for fixed Γ and as $Ra \to \infty$, our numerical solutions converge back to the Chini-Cox asymptotic solutions with $Nu \sim Ra^{1/3}$ and $Re \sim Pr^{-1}Ra^{2/3}$.
[TA02211627]

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Estimating Fine-scale Ancestry in East Asia

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University of Colorado Denver [Mentor:Audrey Hendricks]

Abstract of Report Talk: One type of summary genetic data is genetic data that is pooled across many individuals into allele frequencies. Summary genetic data has high utility in research because it can be easily accessed by the research community. There are multiple large publicly available resources with summary genetic data. For example, gnomAD is a resource that assembles exome and genome sequencing data from a variety of large-scale sequencing projects that is available for the scientific community. GnomAD v3 has 76,156 genomes. There are different ancestral populations represented in gnomAD. Some of the gnomAD groups are admixed populations, meaning they contain multiple continental reference ancestries (e.g. African-American, Latinx groups). Population structure refers to differences in allele frequencies between populations and subpopulations. Population structure needs to be corrected for in most genetic analyses to avoid confounding. Yet, one of the barriers of publicly available genetic summary data is that it is hard to determine the underlying population structure. Summix is a new and efficient method that identifies, estimates, and adjusts for the proportion of continental reference ancestry in publicly available summary genetic data (e.g., in gnomAD). Summix currently uses reference data from 5 ancestry groups to obtain estimates of continental ancestry within summary data. These continental groups are African, Non-Finnish European, East Asian, Indigenous American, and South Asian. However, even within these continental ancestry groups, there is a finer scale population substructure (e.g., Chinese Dai, Han Chinese, Japanese, etc in East Asia). Therefore, the next step is to evaluate Summix's utility to estimate fine-scale ancestry. Determining fine-scale ancestry may help to better adjust for population substructure and give researchers helpful information about what fine-scale populations are in summary data. Here, I test the performance of Summix in estimating fine-scale ancestry in the East Asian group from gnomAD v3.I use East Asian subpopulations from the 1000 Genomes Project and the Human Genome Diversity Project as reference populations to determine fine-scale ancestry in the gnomAD East Asian population

[TS05135902]

[Joint with Katie Marker, Adelle Price, and Audrey E. Hendricks] Received: August 18, 2021

Unitary Representation Theory of Almost Abelian Lie Groups

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University of California Santa Barbara [Mentor:Zhirayr Avetisyan]

Abstract of Report Talk: An almost Abelian Lie group is a non-Abelian Lie group with a co-dimension one Abelian subgroup. In this project we explicitly construct the irreducible unitary representations of connected almost Abelian Lie groups using the Mackey machine. For a simply connected almost Abelian Lie group G with a normal co-dimension one Abelian subgroup N, we find and classify the stabilizer subgroups G_ν of the conjugate G-action on irreducible unitary representations ν of N. Irreducible unitary representations of G are then constructed by the induction procedure from G_ν to G based on the so-called little groups $H_\nu = G_\nu \cap G/N$. It is known that every irreducible unitary representation of G is equivalent to one of this form. We can get a connected almost Abelian Lie group by taking the quotient of a simply connected almost Abelian Lie group with a discrete central subgroup. We use this fact and the representations constructed for simply connected groups to construct the irreducible unitary representations of connected almost Abelian Lie groups. **[TC03164216]**

[Joint with Abby Brauer, Jimmy Morentin] Received: August 17, 2021

Adapting SINDy for Data-driven Discovery of Dynamical Systems

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University of California, Santa Barbara [Mentor:Sui Tang]

Abstract of Report Talk: Multi-agent systems have found wide applications in science and engineering ranging from opinion dynamics to predator-prey systems. A fundamental, challenging problem encountered in these areas is to reveal the link between the collective behaviours and interaction laws. In this report, we leverage recent advancements made within the mathematical modeling community and on sparsity-promoted algorithms in machine learning to consider the data-driven discovery of multi-agent systems with non-local interactions laws dependent on pairwise distances. We adapt the "Sparse Identification of Nonlinear Dynamics" (SINDy) algorithm to approximate interaction laws on systems of interacting agents and demonstrate the effectiveness of the algorithm in various cases. **[TR06023705]**

[Joint with Dongyang Li, Shelby Malowney] Received: August 17, 2021

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[Mentor:]

Abstract of Report Talk: **[]**

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Linear Preservers of Eigenvalues Induced by the Two-Dimensional Ice Cream Cone

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UC Santa Barbara [Mentor:Maribel Bueno]

Abstract of Report Talk: Given a real $n \times n$ matrix A and a closed convex cone $K \subseteq \mathbb{R}^n$, the eigenvalue complementarity problem generalizes the eigenvalue problem by finding λ and nonzero x such that $K \ni x \perp (A - \lambda I)x \in K^*$, where K^* is the dual cone of K. We say λ is a Lorentz eigenvalue of A when K is the Lorentz cone, also called the ice cream cone. Linear maps on the space M_n (resp. S_n) of (resp. symmetric) $n \times n$ matrices preserving Lorentz eigenvalues are well studied in the literature when $n \geq 3$, assuming the map is of the form $A \mapsto PAQ$ or $A \mapsto PA^TQ$ for some matrices P and Q, which we call standard linear maps. Because the Lorentz cone for $n = 2$ is polyhedral, this case has been left out of the literature and turns out to be more complex than $n \geq 3$ in certain ways. In our research, we fully characterize the standard linear maps that preserve the Lorentz spectrum on M_2 and S_2 and show that it is precisely these maps that preserve the nature of the Lorentz eigenvalues, i.e. whether they correspond to eigenvectors on the interior or boundary of the Lorentz cone. In the case of M_2, we show that different kinds of linear preservers are possible compared to M_n for $n \geq 3$. Moreover, we prove that all linear maps preserving the Lorentz spectrum on S_2 are standard and conjecture that the same holds for M_2. **[VJ05181444]**

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Ratio of Set Orbits of Permutation Groups and Their Order

Zili Wang (ziliwang271@berkeley.edu)
Texas State University [Mentor:Yong Yang]

Abstract of Report Talk: A permutation group G acting on a finite set Ω induces a natural action on the power set $P(\Omega)$. Let G be a permutation group of degree n that do not contain A_l, $l > 4$, as a composition factor, and let $s(G)$ denote the number of orbits of G on $P(\Omega)$. In this presentation, we determine $\inf(\frac{\log_2 s(G)}{\log_2 |G|})$. **[WZ05121444]**

[Joint with Matthew Kortje, Michael Gintz, Maggie Laurence]

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On the conjugacy problem in a variant of the well known Thompson Group

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The Ohio State University [Mentor:Sergei Chmutov]

Abstract of Report Talk: The well-known "Thompson Group," which we denote F_2, is the group of continuous, piecewise-linear self-homeomorphisms of the $[0, 1]$ interval, which are differentiable everywhere except a finite set of dyadic rationals (rational numbers whose denominator is a power of 2), and such that, where it exists, the derivative is always an integer power of 2. By changing this definition so as to keep everything the same but with threes instead of twos, we arrive at an object we denote F_3.

In their 2008 paper, Belk and Matucci solve the conjugacy problem for F_2 using so-called "strand diagrams". In his 2018 paper, Jones demonstrates connections between F_2 and F_3 in terms of "tree diagrams" which are closely related to strand diagrams. In this presentation we generalize Belk and Matucci's methods to solve the conjugacy problem in F_3. **[WJ05185309]**

[Joint with Dr. Matt Harper, Dennis Sweeney, Torey Hilbert, Alexander Patterson] Received: August 17, 2021

Glacial Cycle Attractors

Leah Woldemariam (lsw85@cornell.edu)
Cornell University [Mentor:Alice Nadeau]

Abstract of Report Talk: Oftentimes, a system must be modeled, but there is a lack of information surrounding the system apart from observed data. Attractor reconstruction methods allow us to create a dynamical system model from a single observed time series. Takens theorem allows us to create a topologically equivalent attractor of the dynamical system from this single observed time series. The data used in the reconstruction and the reconstruction itself can be analyzed with various techniques such as computing fractal dimensions or measuring dynamical invariants. Using attractor reconstruction methods and time series analysis, we investigate the relationship between variables commonly used in conceptual glacial models and compare the ability of different data sets to accurately reconstruct the glacial cycle attractor. **[WL31234835]**

[Joint with Alice Nadeau] Received: August 18, 2021

On the number of certain subgraphs of the unit distance graph

Dora Woodruff (dorawoodruff@college.harvard.edu)
Baruch College [Mentor:Nora Frankl]

Abstract of Report Talk: The Erdős Unit Distance Problem, which asks for the maximum number of times that the unit distance can occur among n points in the plane or $\mathbb{R}^3$, is one of the most well-known problems in Discrete Geometry. Many variants have been studied in the work of Elekes, Pach, and others. Our work is on bounds on the maximum number of isomorphic copies of a given subgraph in a unit distance graph, the graph whose edges represent unit distances between the n points. Recently, Palsson et al. asked about the case of paths of fixed length. We continue this line of research, proving sharp results on the maximum number of paths, cycles, or copies of a given tree in the unit distance graph on a specific sphere. Interestingly, while for the planar path problem the answer depends on the length of the path mod 3, on the sphere it depends on the length mod 5. We also find bounds on the number of copies of any given 3-regular graph in $\mathbb{R}^3$ by analyzing a linear optimization problem we derive from the Cutting Lemma. Our proof is inspired by a paper of Agarwal and Sharir on unit simplices. **[WD30164143]**

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The Picard group of a general toric variety in higher dimensions

Xiaorun Wu (xiaorunw@princeton.edu)
Princeton University [Mentor:Chenyang Xu]

Abstract of Report Talk: Suppose $\{v_1, \cdots, v_r\}$ is a finite set of vectors in $\mathbb{R}^d$. Denote

$$\sigma = \{x \in \mathbb{R}^d : x = \lambda_1 v_1 + \cdots + \lambda_r v_r, \lambda_i \in \mathbb{R}, \lambda_i \geq 0\}$$

to be the polyhedral cone. We say σ is a rational cone if all generators v_i belong to $\mathbb{Z}^d$, and σ is strongly convex if $\sigma \cap (-\sigma) = \{0\}$.

Next, define a rational fan Δ in $\mathbb{R}^d$ as a finite union of rational cones such that:

- every cone of Δ is a strongly convex, polyhedral, rational cone,
- every face of a cone of Δ is a cone of Δ,
- if σ and σ' are cones of Δ, then $\sigma \cap \sigma'$ is a common face of σ and σ'.

Additionally, we say Δ is complete if it covers $\mathbb{R}^d$, i.e. $\bigcup_{\sigma \in \Delta} \sigma = \mathbb{R}^d$. For each complete rational Δ, we may associate a toric variety $T_N(\Delta)$, which we denote as X. Finally, denote the Picard group of X as $\mathrm{Pic}(X)$.

The relationship between $\mathrm{Pic}(X)$ and combinatoric property of Δ have been extensively studied. Ford and Stimets have shown in 2002 that, if $\Delta \in \mathbb{R}^3$ is a complete rational fan such that every 3-dimensional cone in Δ is non-simplicial, then in any non-empty open neighborhood of Δ there exists a fan Δ' such that every Δ'-linear support function is linear and the Picard group of the associated toric variety is zero.

In this paper, our contribution is two-fold: first, we present a simpler proof of Ford and Stimets's original results. Then extending their results, we study the case in when Δ is a complete rational fan in $\mathbb{R}^d$ such that every three-dimensional cone in Δ is non-simplicial for some $d \geq 4$. **[WX06162913]**

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The Picard group of a general toric variety in higher dimensions

Xiaorun Xiaorun Wu Wu (xiaorunw@princeton.edu)
Princeton University [Mentor:Chenyang Xu]

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Simulate flocking behavior with residual network

Siwei Xu (siwei.xu@emory.edu)
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Ohio State University [Mentor:Yulong Xing]

Abstract of Report Talk: Deep neural networks have been demonstrated to be powerful tools for data driven modeling. In this talk, we explore the use of residual networks (ResNets) for simulations of flocking phenomenon for population dynamics. We use Cucker-Smale model, an autonomous ordinary differential equation model, to generate bird trajectories and apply them as observation data for the neural-network training. The ResNets are constructed by following the recent work of Qin, Wu and Xiu in 2018. We successfully train ResNets to approximate the Cucker-Smale model and simulate the flocking behavior of birds. Different ways to generate training data have been explored and studied. We also include a parameter β, which describes the strength of the impact a bird has on its neighbor, into the training process to study the flocking behavior of different settings. A successfully trained neural network model can be used as a predictive tool and offers flexibility to further study the underlying process. **[XS04101603]**

[Joint with WeiHung Su, Zheng Sun, Joseph Hunter] Received: August 17, 2021

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[Mentor:]

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Effective Redshift for Brown-Peterson Spectra

Tristan Yang (tristanyang@college.harvard.edu)
Harvard University [Mentor:Elden Elmanto]

Abstract of Report Talk: The classical Quillen-Lichtenbaum conjecture relates the values of zeta functions with the sizes of certain algebraic K-theory groups. Results of Quillen and Thomason reduce this to the question of whether a certain localization map on K-theory is an isomorphism in large degrees, now proved by Voevodsky, Rost, and others. Inspired by this, Rognes has subsequently proposed a "higher chromatic" version of the Quillen-Lichtenbaum conjecture, viewing the Quillen-Lichtenbaum conjecture as an instance of "redshift in chromatic heights." This was recently verified in many cases by Hahn and Wilson. We make the result of Hahn and Wilson *effective*. In particular, we make explicit the range in which the relevant localization map is an isomorphism, and show that it is the same as the classical setting in many cases. **[YT06165006]**

[Joint with additional mentor: Tomer Schlank, Hebrew University of Jerusalem] Received: August 18, 2021

Winning Strategy of Two-Player, Multiplayer and Multialliance Generalized Zeckendorf Game

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Williams College [Mentor:Thomas Martinez]

Abstract of Report Talk: Zeckendorf proved that every positive integer n can be written uniquely as the sum of non-adjacent Fibonacci numbers; a similar result holds for many other positive linear recurrence sequences. These legal decompositions can be used to construct a game that starts with a fixed integer n, and players take turns using moves relating to a given recurrence relation. The game eventually terminates in a unique legal decomposition, and the player who makes the final move wins. Previous work on the Fibonacci game proved that player 2 always has a winning strategy for $n > 2$, though the proof is non-constructive and thus it is still open how to play optimally.

The Generalized Zeckendorf game is also known as the (c, k)-nacci game. The (c, k)-nacci sequence is defined as $S_1 = 1, S_{i+1} = cS_i + cS_{i-1} + \cdots cS_1 + 1$ for any $i \leq k$, and $S_{i+1} = cS_i + cS_{i-1} + \cdots cS_{i-k}$ for any $i > k$. Building on earlier work, we focused on proving who has a winning strategy for the 2-player, multiplayer and multialliance Generalized Zeckendorf games. We give a non-constructive proof that for the (c, k)-nacci game, when $k \geq 3$, player 1 has a winning strategy when c is even and player 2 has a winning strategy when c is odd; the proof involves parity arguments and an analysis of game states through associated graphs. Interestingly, for the $(1, 2)$-nacci game, if player 2 makes an incorrect chocie for their first move then player 1 can win, though we do not know what an incorrect move is. Both of these proofs are non-constructive, and finding the specific constructive winning strategy remains an open problem. Furthermore, we proved that for the generalized Zeckendorf game with players $p \geq c + 2$, no player has a winning strategy for any $n \geq 3c^2 + 6c + 3$. We were able to find a stricter lower boundary, $n \geq 7$, in the case of the 3-player $(1, 2)$-nacci game. Then we extended the result from the multiplayer game to multialliance games, showing which alliance has a winning strategy or when no winning strategy exists for some special cases of 2-alliance and multialliance games. **[YJ04123135]**

[Joint with Steven J Miller (Research Program Advisor)] Received: August 17, 2021

Effective Nullstellensatz over Finite Fields

Jeffery Yu (jeffery@mit.edu)
Baruch College [Mentor:Guy Moshkovitz]

Abstract of Report Talk: Hilbert's Nullstellensatz is a central theorem in classical algebraic geometry, relating ideals of polynomials over algebraically closed fields to their locus of common zeros. Kollar gave sharp effective bounds for this in 1988. We study finite field analogues of the Nullstellensatz, where sharp effective bounds are not yet known. In their paper introducing connections between bias and rank, Green and Tao applied their result to prove an effective Nullstellensatz for finite fields. However their bounds were of Ackermann-type. We apply recent improvements to the bias-vs-rank inequality to improve the bound to tower-type. We also present a new constructive proof of the finite field Nullstellensatz that achieves linear bounds. This construction avoids the rapidly-growing regularization step typically used alongside the bias-vs-rank inequality, and thus provides a close-to-optimal bound. [YJ06103816]

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